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The Logical Foundations of Science

W. E. Johnson

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LOGIC

PART III

THE LOGICAL FOUNDATIONS OF SCIENCE

BY

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DOVER PUBLICATIONS, INC.
NEW YORK

Nyack College Library
Nyack, NY 10960
86381

Published in Canada by General Publishing Company, Ltd., 30 Lesmill Road, Don Mills, Toronto, Ontario.

Published in the United Kingdom by Constable and Company, Ltd., 10 Orange Street, London WC 2.

This Dover edition, first published in 1964, is an unabridged and unaltered republication of the work first published by the Cambridge University Press, Part I in 1921, Part II in 1922 and Part III in 1924. This edition is published by special arrangement with the Cambridge University Press.

#373125

Standard Book Number: 486-21175-4
Library of Congress Catalog Card Number 64-18362

Manufactured in the United States of America
Dover Publications, Inc.
180 Varick Street
New York, N. Y. 10014

CONTENTS

INTRODUCTION

PAGE

§ 1. Review of the lines of discussion occupying Parts I and II. Pre-mathematical Logic xiii

§ 2. Summary of the author's treatment of Mathematical Logic . . . xiv

§ 3. The ontological topics introduced in Part III: comparison with the treatment accorded to these topics by other logicians. The place of postulates in inductive theory xvi

§ 4. The division of existents into continuants and occurrents . . . xviii

§ 5. The dualistic view. Meanings of 'of' xix

§ 6. Examination of the distinction between occurrent and event: the former being identified and discriminated by reference to difference of adjectival determinable, the latter by reference to difference of location . xxi

§ 7. Agent and Patient. Immanent and Transeunt. Monadism and Monism xxiii

§ 8. Combination of Immanent with Transeunt Causation xxv

§ 9. Interaction between 'mind' and 'body.' Parallelism and Correspondence. Parallelism and denial of Causality. Psychical and Physical processes presented in cycles xxvi

§ 10. One-sided correspondence. Impartial Dualism xxviii

§ 11. Attempt to meet attacks upon Impartial Dualism xxxi

§ 12. Invariability and Causality xxxii

§ 13. Misrepresentations of the deterministic position xxxiii

§ 14. Explanation of the necessity for introducing the discussions of psychological and metaphysical topics into Logic xxxv

CHAPTER I

FACT AND LAW

§ 1. Statements of fact 1

§ 2. Reduction of statements of fact to the form: A certain *P* is *p* . . 2

§ 3. Verbal expression for the distinction between the universal of fact and the universal of law 4

§ 4. Resumé of uses of the term 'possible' 6

§ 5. Inadequacy of the purely factual interpretation of certain types of proposition 11

§ 6. The 'possible' and the 'hypothetically necessary' 14

CHAPTER II

THE CRITERIA OF PROBLEMATIC INDUCTION

PAGE

§ 1. Instantial propositions as premisses of induction 16
§ 2. The 'course' of nature and the 'laws' of nature 17
§ 3. Complementary enumeratives and complementary universals . . 19
§ 4. Requirement of maximum variety amongst examined instances . . 21
§ 5. Combination of variety and similarity 24
§ 6. Number and proximity as substitutes for variety and similarity . . 24
§ 7. Independence of characters and incongruence of instances . . . 25
§ 8. First formulation of the criterion for the Method of Complementaries. 27
§ 9. Criterion of precision 27
§ 10. Complex comprehensive exactitude 28
§ 11. Tabular schematisation corresponding to Bacon and Mill's empirical generalisation 28
§ 12. Discussion of the relation between 'hypothesis' and 'generalisation' . 30
§ 13. The degree of probability varies directly with the degree of ascertained accordance 33

CHAPTER III

DEPENDENCY AND INDEPENDENCY

§ 1. Separation in thought between the determining and the determined characters—these being presented as merely conjoined in fact . . 36
§ 2. In experimentation the determining factors are known *before* and the dependent factors only *after* the result of the experiment . . . 38
§ 3. Errors in observation (unaided by experiment) resulting from the false supposition that certain factors are independent which are in reality dependent 40

CHAPTER IV

EDUCTION

§ 1. Inference (so-called) from particulars to particulars should be termed *eduction* 43
§ 2. Eduction involves a minor, middle and major premiss containing not only a minor and a major term but also two middle terms respectively intensional and extensional. Schematisations of eduction . . . 44
§ 3. The commonly alleged distinction between induction and analogy should be replaced by the distinction between the two mediating terms respectively extensional and intensional required for eduction and for generalisation. Cases tabulated where all the evidential data are *in favour of* a certain educed conclusion 46

PAGE

§ 4. In estimating the evidential force of instantial data, characters must be counted only so far as they constitute an independency, and instances only so far as they constitute a variancy 48
§ 5. Further explication of the above principle 48
§ 6. Further exposition of the principle as regards evidential premisses without respect to inferred conclusion 50
§ 7. The nature of the inferred conclusion in its relation to the evidential premisses 51
§ 8. Irreducible contrast between 'subject' and 'predicate' . . . 52
§ 9. The above contrast as corresponding to the fundamental distinction between separation and discrimination 53

CHAPTER V

PLURALITY OF CAUSES AND OF EFFECTS

§ 1. Incomplete assignment of Cause yields alternative Effects just as incomplete assignment of Effect yields alternative Causes . . 54
§ 2. An assignment of Cause or of Effect is *complete* only in reference to the Effect or to the Cause 56
§ 3. A conjunction of cause-factors constitutes a completed cause; a conjunction of effect-factors constitutes a completed effect . . . 57
§ 4. Plurality means that *some but not all* of the possible values of a determinable may be put into the subject-terms of the universals in which the same determinate character is predicated 59
§ 5. Correction of the demonstrative figures of induction required by the consideration of a possible plurality of causes or of effects . . . 60
§ 6. Reversibility of the universal proposition connecting the completed cause and the completed effect 62
§ 7. The step-by-step process by which the completed cause and the completed effect are reached 63

CHAPTER VI

CAUSE-FACTORS

§ 1. Preliminary exposition of the notion of change in its reference to a Continuant 66
§ 2. Continuant as Cause must be distinguished from Occurrent as Cause. Cause and Effect are coordinate when interpreted as Events and each is reciprocally inferable from the other. Nevertheless the *temporal* relation is regarded as *not* reciprocal 68
§ 3. Alleged distinction between two types of objective law . . . 70
§ 4. The *properties* of continuants regarded as causal 71
§ 5. Effects—like causes—not merely resolvable into events or occurrences 72
§ 6. Temporal sequence of cause-occurrence and effect-occurrence explained in reply to philosophical criticism 74

CHAPTER VII

THE CONTINUANT

PAGE

§ 1. The fundamental notion of causal connection between *movements* that occur in space entails reference to a physical continuant conceived as *that which moves* 78

§ 2. The term 'continuant' is chosen to replace 'substance' in order to free the notion from certain philosophical implications inseparable from the latter, and to emphasize the inevitable residuum which (as maintained by the writer) is indispensable for science. Thus, in the first place, the extension of continuance to an infinite future and an infinite past which is often attributed by philosophers to *substance* may be dispensed with in the scientifically conceived *continuant* 80

§ 3. Secondly, substantival continuance does not necessarily entail any adjectival changelessness 80

§ 4. Thirdly, the ultimate substantival continuant is not necessarily simple. Moreover, the existential components which may constitute a continuant-unity may not themselves be continuant 81

§ 5. What in general holds of the physical continuant holds also of the psychical continuant; but the structure of the latter exhibits far higher complexity than the former. For, whereas motion is the sole fundamental mode of manifestation of the physical continuant, there are many irreducible but interconnected modes of manifestation to be attributed to the psychical continuant. Within each of these several modes the conception of change must be separately applied 82

§ 6. The conception of change, moreover, involves the replacement of one by another manifestation, of which different determinate characters under the same determinable may be predicated. 84

§ 7. The mutually implied conceptions of substance and causality (in their residual scientific significance) lead to the notion of *property*, which is the appropriate adjective characterising a continuant as contrasted with an occurrent. A property must be conceived as a defined potentiality: other adjectives are conceived as descriptive of actualities . . . 86

§ 8. The comparatively primitive attempts to systematise the manifold of reality illustrate the same principles and postulates which govern the procedure of advancing science 89

§ 9. Sub-continuants and sub-occupants 92

§ 10. Causality within the manifestations of a single continuant . . . 93

§ 11. Uniformities embracing *different* continuants 94

§ 12. The unity of a continuant exhibited in causal formulae. Alterable and unalterable properties 95

§ 13. Comparison of the views here maintained with those of Kant . . 98

§ 14. Fundamental contrast between the author's views and those widely current since Hume and at the present day 100

CHAPTER VIII

APPLICATION OF CAUSAL NOTIONS TO MIND

PAGE

§ 1. The effect of purposive control in modifying the more mechanical mental processes points to a form of causality—operating within the experience of a single individual—which is closely analogous to the form of causality termed transeunt 102

§ 2. If there is any direct determining influence of the psychical upon the physiological or conversely, such influence undoubtedly comes under the head of transeunt causality. If, for example, we assume that sensations and other quasi-mechanical mental processes are directly determined by neural processes, there will be a direct correlation between such mental processes on the one hand and the neural processes on the other. If, further, we assume that the phases of feeling and cognition are *partially* determined by sensations, then there will be a partial—but indirect—correlation between the former and the latter. But such correlation must be limited to those variations which can be said to *correspond* with one another. And it would appear that there are no variations in neural process which could correspond to the variations of feeling and cognition 104

§ 3. Any felt effort or strain that may be incurred by mental activity has both a physiological and a sensational aspect which can properly be said to correspond. But so far as the effects of such effort are *intended*, we have reasonable ground for asserting the operation of transeunt causality, in which the causal agency is psychical and the direct effect physiological 107

§ 4. The presumption that foreknowledge operates in the psychical determination of physical effects, is tantamount to attributing real causal efficiency to such foreknowledge. But if this foreknowledge could be reduced to merely physiological terms, mental causality would be an illusion 109

§ 5. Mental activity assumes two distinct types. (1) Motor activity which produces sensational effects, and (2) The activity of attention which produces cognitional effects. In both cases, grounds are put forward for maintaining that the causal agency entailed is purely psychical . 110

§ 6. Explicit grounds for the contention that changes of cognitive phase have no counterparts in the changes of neural process 113

§ 7. The prevalent confusion between images and ideas largely accounts for the refusal by physiologists and psychologists to accept the above view 116

§ 8. An analysis of the phases of consciousness accompanying reflex processes best illustrates the contrast between physiological and psychical causal agency 117

§ 9. But an analysis of conative conflict yields the most important indication that psychical agency is really causally operative 120

§ 10. The cognitive aspects of deliberative process complete the grounds for our main contention 124

§ 11. 'Judgments of value' stand to 'conations' as cause to effect, not conversely 125

CHAPTER IX

TRANSEUNT AND IMMANENT CAUSALITY

PAGE

§ 1. The event termed 'Movement' cannot be reduced to merely spatio-temporal terms; since it requires something that moves, and that retains its continued identity within the spatio-temporal bounds of the event 127

§ 2. The causality formulated in the first law of motion is wholly immanent, but that formulated in other dynamic laws is essentially transeunt . 128

§ 3. Analysis of the immanent and transeunt factors entering into an elementary physical process 129

§ 4. Contrast to the previous illustration, where cause and effect are reversed. In cases of immanent process, where a cause-occurrent and an effect-occurrent are simultaneous, our ground for deciding which of the two occurrents is cause and which is effect is based upon the principle that that occurrent which is effect in the transeunt process is cause in the immanent process 131

§ 5. In the analysis of emotional experiences, which entail diffused organic sensations, another illustration is afforded of the ways in which transeunt and immanent causality are distinguished and combined . . . 134

§ 6. Fundamental distinctions between the psychical and the physical continuant 136

§ 7. The finally unique distinction between the two 139

§ 8. Formulae which are correctly expressed in terms of immanent causality *as regards unitary wholes* are frequently equally correctly and more adequately expressed in terms also of transeunt causality *as regards constituent parts* 140

CHAPTER X

CONVERGENT AND DIVERGENT CAUSALITY

§ 1. Diagrammatic representation, by the use of parallel, converging and diverging lines, to explain the different forms assumed in causal complexes,—parallel lines being employed to represent causal independence 143

§ 2. Application of above to dynamic and chemical formulae . . . 146

§ 3. Further application to psycho-physical formulae 148

§ 4. Extension of the diagrams to illustrate the more complicated formulae in physics 152

§ 5. Similar exposition of the more complicated forms of psycho-physical causality 156

CHAPTER XI

TEMPORAL AND SPATIAL RELATIONS INVOLVED IN CAUSALITY

PAGE

§ 1. The conception of connectional determination as involving spatio-temporal relations 161
§ 2. Order: discrete and continuous 162
§ 3. The possibility of discontinuous change 165
§ 4. The idea of connectional determination extended to interpsychical causality 169
§ 5. Analogies between the spatially inner and outer on the one hand and the temporally prior and posterior on the other 171
§ 6. Explanation of 'potential' causality 173
§ 7. Examination of certain elementary physical processes . . . 175

APPENDIX ON EDUCTION

§ 1. Notation to be adopted. Two elementary formulae 178
§ 2. The notion of probability. Proposal and supposal. Contrast with the relation of implication 179
§ 3. The two working axioms of the probability-calculus 181
§ 4. Four corollaries from the two axioms 181
§ 5. Necessity for special postulates before the theorems of the calculus can be applied to any actually given problem 182
§ 6. Two postulates are adopted in the proposed establishment of a theory of eduction 183
§ 7. Formal proof of the eductive theorem 184
§ 8. Elucidation of the formula for successive values of *N*. Mnemonic schematisation 186
§ 9. Discussion of the grounds on which the adoption of the two postulates is based and of the type of case for which the postulates are legitimate 187

INDEX 190

INTRODUCTION

§ 1. The subjects discussed in Parts I and II come within the scope of what may be called Formal Logic. Here the proposition is taken to be the immediate object of a possible assertion; and a consideration of its nature leads to the conception of the antithesis and connection of substantive with adjective, as disclosed in the analysis of the simplest articulate form of judgment. The function of language and more particularly of names is examined. It is held that the different forms assumed by compound propositions are indicated by various words, not standing for substantival or adjectival constituents, but expressive of the modes in which simple propositions or their constituents are to be connected by constructive thought. Such considerations lead to a preliminary definition and enumeration of logical categories roughly corresponding to (and replacing) the grammatical enumeration of parts of speech.

In the more detailed examination which follows, substantives proper or existents are distinguished from quasi-substantives, adjectives predicable of the former being termed primary and those predicable of the latter secondary. Modality, in its formal aspects, is treated under the more general heading of secondary propositions. Adjectives are divided into transitive adjectives (otherwise relations) and intransitive adjectives, in precise analogy with the grammatical division of verbs; and again into monadic, dyadic, triadic, etc. according to the number of substantive-terms which are entailed in their employment. A prominent place is given to the distinction and connection, amongst adjectives in general, between adjectival determinables and adjectival determinates. This distinction is utilised in all the further developments of logical theory. The relations between inference and implication, the former of which is essentially epistemic and the latter essentially constitutive are entered into at considerable length. In particular, certain general and fundamental *principles* of inference are laid down and contrasted as formal with the *premisses* of inference which are material.

Inferences and implications are divided into the two species demonstrative and problematic. The term induction has been used, with some hesitation, to include four species—intuitive, summary, demonstrative and problematic. The first three of these are examined in Part II, the fourth being reserved for Part III. Deductive inference or implication is treated in connection with the intuitive foundations of pure logic and pure mathematics; as also with summary induction.

§ 2. It is contended, in agreement with most recent logicians, that Arithmetic and (more generally) Pure Mathematics develops from Pure or Formal Logic: i.e. that the conceptions and axioms underlying the former are none other than those underlying the latter. If any distinction is to be made between Pre-mathematical Logic and Pure Mathematics it is suggested that the latter introduces certain adjectives and relations which in the strictest sense are constant, i.e. represented by words or symbols of which it is essential for the science that the meanings should be understood in one invariable sense; whereas the intelligent apprehension of pre-mathematical formulae requires that symbols for adjectives and relations in general should be understood merely illustratively to stand indifferently for any actual adjectives that might be substituted for them.

Now, in the transition from pre-mathematical to mathematical logic, the first notions that demand explicit recognition are those of *identity* and (its contrary) *otherness* or diversity. These two relations are applicable to any entities whatsoever coming under any category whatever. Thus if *a* unambiguously denotes any entity whatever and *b* unambiguously denotes any entity whatever, then (so far) the entity denoted by *a* may be identical with and may be other than that denoted by *b*. At this point, the two axioms that identity and otherness are co-alternate and co-disjunct have to be explicitly formulated. Speaking loosely, the relation of identity yields the notion of *one* and that of otherness yields the notion of *two*. More accurately and precisely the conception of *number* is developed from that of a certain sub-division of the genus *relation* termed *one-one*; and *one-one* relations are defined entirely in terms of identity and otherness; i.e. no other notions than these are involved beyond those appertaining to pre-mathematical logic. In this way, the definition, not only of any assigned finite number, but even of infinite number introduces (besides pre-mathematical notions) *identity* and *otherness* alone. In the higher branches of arithmetic other relations, dyadic, triadic, etc., are introduced, especially those which develop from the general notion of *order*; and these are all expressed and defined in terms of words or symbols having a fixed invariable meaning that must be understood by the mathematician as such.

Not only must the mathematician understand the meanings of the constant symbols introduced and defined in the science, but also his intelligent assent is required to be given to certain axioms (or primarily fundamental propositions) expressed in terms of these symbols; and his intelligence must be further exercised in following the demonstrative procedure by which derivative formulae are progressively inferred. He discovers, not only the comparatively unimportant fact that the conclusions are true provided that the originally premised axioms are true, but also the more important fact that the conclusions follow demonstratively from a judicious combination of these axioms and *these alone* —none other being required. The account of symbolism and allied topics in Part II includes references to processes of thought and thus is largely psychological—in this respect differing from the accounts given by professedly formal logicians.

§ 3. Part III opens new ground. Such ontological conceptions as those of substance and causality—even of 'matter' and 'mind'—are explicitly introduced and their significance discussed in detail. In this way, a claim is made that logic should be recognised as a department of philosophy in a higher sense than any warranted by the restriction of its scope to what has been termed formal logic. It is true that inductive logicians have bestowed much care upon the examination of the nature of cause and, less explicitly, of substance. But for the most part they have deliberately excluded any discussion of the philosophical implications attached to these notions; either on the ground that these implications belong to metaphysics or that they are to be rejected *in toto* as merely bad metaphysics. For example, though much of what Mill has said and Venn has said better about causal and other uniformities has its value, yet it is obvious that their treatment gives us no instruction on the philosophical questions at issue. Moreover, not only the professedly philosophical logicians but, strangely enough, also the humbler inductive logicians have overlooked or devoted insufficient attention to many methodological problems the discussion of which belongs to the logic of the sciences. This constitutes my apology for entering with considerable detail into topics which lie on the borderland between Logic as Methodology and Logic as Philosophy.

The inductive logicians may be said to have presented a philosophical case only on the supposition that they are to be interpreted as having contended for the inutility of such notions as those of causality and substance in the establishment of scientific generalisations. Thus Mill's reduction of the causal relation to invariable and unconditional sequence is naturally interpeted as tantamount to the rejection of the notion of cause in any philosophical sense. And this is certainly the contention of those among later empiricists who have concerned themselves with the problems of scientific induction. In fact, the more modern view expressly held by formal logicians of the present day (who are mostly empiricists of the school of Hume) is that all the principles of induction (with the doubtful exception of probability) are derivable by an extension of the principles of deduction much as Pure Mathematics is a mere extension of Pure Logic. With this view I am in partial agreement, and the discussions of Part III are largely concerned with the points both of agreement and disagreement between my view and that of the more extreme empiricists.

In examining the logical foundations of science, I have found it impossible to separate the Epistemological (or preferably Epistemic) from the Ontological point of view. The explanation of this impossibility is that, as it appears to me, certain notions—and certain propositions expressible in terms of these notions—must be *postulated*, if science is to be validly established.

By a postulate I understand a proposition that is assertorically and not merely hypothetically entertained; but yet is adopted neither on the ground of intuitive self-evidence nor of inductive confirmation. More positively, a postulate is framed in terms not given in experience, and these terms enter even into the instantial propositions which are problematically universalised by induction. Postulates, in my view, enter even into mere observations of instances which may otherwise be termed judgments of perception. In these judgments the thinker predicates not merely a concomitance of characters presented to him; but, besides concomitance, causality; and, besides presentment, reference to substance.

§ 4. The ontological discussions of Part III are centred upon the recognition of the two concepts, causality and substance. But I have discarded the term 'substance,' for reasons which need no enumeration, in favour of the term 'continuant.' The genus 'substantive proper,' otherwise termed 'existent,' is divided into the two species 'Continuant' and 'Occurrent.' The distinction among substantives between continuants and occurrents plays a similarly prominent part in material logic as is played in formal logic by the distinction among adjectives between determinables and determinates. But no *analogy* can be drawn between the antithesis or connection in the one case and that in the other. Negatively, it may be said that a continuant is not a mere collection of occurrents just as a determinable is not a mere collection of determinates. Further than this we can only say that a plurality of occurrents is constructed by thought into a unity by virtue of the nexus of causality and a plurality of determinates by virtue of the relation of opponency or incompatibility. No *positive* analogy can be drawn, owing (it would seem) to the ultimately irresolvable antithesis between substantive and adjective.

§ 5. A more detailed summary of the views propounded in Part III on ontological problems may now be given.

In the first place, I have adopted the dualistic position which recognises a fundamental distinction between the psychical and the physical, and attributes reality to both in the same unequivocal sense. Whether or not the view is philosophically tenable, at any rate any examination into the principles of science would seem to be impossible without some such hypothesis as that of dualism. Spinoza's acceptance of two unsynthesised attributes,—thought and extension—illustrates, in more or less veiled guise, the very same fundamental position as that adopted by the dualist. But the view that I wish to put forward is less dualistic than Spinoza's, in that I profess to present the psychical and the physical in some sort of synthesis with one another, and not in mere unreconciled antithesis. What I hold to be important in the dualistic position is the recognition of two kinds of *agency*—psychical agency and physical agency. Of my views, on this and kindred matters, I do not profess to be able to offer any direct demonstration, nor do I believe that my philosophical opponents can offer any valid refutation. The more detailed exposition of my philosophy must be allowed to be taken as a substitute for strict demonstration.

A continuant is defined to be that which continues to exist throughout some limited or unlimited period of time, during which its inner states or its outer connections with other continuants may be altering or may be continuing unaltered. In the first place, then, the continuant must be contrasted with its states—the possessive pronoun here pointing to a unique species of 'tie' indicated by the preposition *of* to be understood in a specific sense differing from all other senses. There is no relational word (as far as I know) that can be used to express this specific meaning of 'of,' parallel to the relational word *characterising* which expresses the specific meaning of 'of' in such a phrase as "the quality *of* this or that." In fact, the two meanings of the word are continually combined in constructions such as those expressed by the phrase "the quality *of* this or that state *of* this or that continuant." Just as a quality must be attached or referred to this or that state, so a state must be attached or referred to this or that continuant. We may also speak of a property *of* this or that continuant to mean a property *characterising* this or that continuant, so that *property* (in this application) is a species of the genus *adjective*.

Now while we cannot say that a *continuant* occurs, we *can* say that a *state* occurs; and anything that may be said to occur will be called an 'occurrent.' And I lay it down that any occurrent must be referred to a continuant or to two or more connected continuants. The reference of an occurrent to connected continuants will be entailed when we speak of *transeunt* causality; while the reference of an occurrent to a single continuant will be entailed sometimes in speaking of *immanent* causality and sometimes in speaking of *transeunt* causality.

§ 6. In many applications 'occurrent' and 'event' may be taken as synonyms; but, properly speaking, they must be distinguished. Thus what is called a *single* event is (or may be) resolvable into a plurality of occurrents of different kinds. The resolution of an event into a plurality of occurrents must not be confounded with the partition of an event into a plurality of parts. The parts of an event are themselves events; and these are distinguished from one another by their difference of spatio-temporal location. On the other hand, the occurrents composing an event cannot be distinguished by difference of location, for they must be located within the same spatio-temporal boundaries as the event itself.

The above general account of the distinction between occurrents and events may be considered first in regard to physical and next in regard to psychical events. A physical event has a spatio-temporal extension which is defined by the spatio-temporal boundary within which it falls, which again determines the four-dimensional magnitude of the extension. In order to distinguish between one and another physical event it would seem, therefore, both necessary and sufficient that we should be able to assign different spatio-temporal boundaries to the two. This holds even of the event-parts of a whole event as distinguished from one another and from the whole; the different event-parts being said to occupy different parts of the extension occupied by the whole event. Now, besides mentally dividing an event into parts, we may also mentally resolve an event into occurrents. The several occurrents which thus compose an event are distinguished, not by the spatio-temporal position which they occupy, but by the different adjectival determinables under which their determinate characters fall. Now all that is here said about physical events and physical occurrents holds also of psychical events and psychical occurrents, except for the fact that spatial reference cannot be applied to the latter and temporal reference only remains. It follows that the extension of a psychical event and the magnitude of its extension are one-dimensional instead of four-dimensional. Hence, whereas difference of position would seem to be necessary and sufficient to mark off one physical event from another, difference of dating is not necessary or sufficient for marking off one psychical event from another. Thus, if one person is suffering tooth-ache contemporaneously with another person's reflecting upon a mathematical problem, we should speak of these as *two* events, although we cannot attribute to either of them spatial extension or boundary and, therefore, cannot attribute to them *different* spatial extensions or boundaries.

This shows that in order mentally to separate one psychical event from another we must postulate, not only a difference of temporal position (if any), but also different psychical continuants to which the two different psychical events are to be referred. *A priori*, indeed, the same must hold as regards physical events; i.e. two simultaneous events might occupy the same locality, which is tantamount to the possibility that two bodies (physical continuants) should be 'occupying' the same place at the same time. This postulate would be necessitated if we found that two phenomena, not in immediate causal relation, such as pressure and attraction were occurring at the same place and at the same time; just as we are necessitated to postulate two psychical continuants when two psychical events, not in immediate causal relation, occur within the same period of time.

§ 7. In transeunt causality, as so far expounded, we conceive two continuants—which in the first instance are to be physical—in causal connection with one another; in such wise that the alterable 'state' of the one continuant is attributed as effect of its alterable relation with the other. This conception of transitive causality gives significance to the antithesis 'agent-patient.' That continuant whose 'state' is occasioned by its relation with the other continuant is termed (in this connection) patient, and that continuant whose relation to the former occasions the state is termed agent. Logicians who have rejected the antithesis between agent and patient have done so on the ground that every agent is at the same time patient and every patient is at the same time agent. But, even, if this were universally the case, the distinction would remain; since the state of the one continuant is effect of its relation with the other continuant while the concurrent state of the other continuant is effect of its relation with the former. We can always distinguish between the one cause which occasions *its* effect and the other cause which occasions *its* effect. Hence, I should substitute for Kant's three categories of relation: Continuant and State; Cause and Effect; Agent and Patient.

Several points in the consideration of transeunt and immanent causality must be noted.

(*a*) Processes which are immanent to a whole system of interacting continuants may always be regarded as entailing transeunt causality between the parts of the whole system. This aspect of causality is familiar to the student of Physical Science. Or—to express the same principle in converse form—if we primarily conceive of interaction between parts of a system as exhibiting transeunt causality, we may (without contradiction) express our formulae in terms of causality immanent to the whole. Physics is at first provisionally monadistic, but it becomes increasingly monistic, in the sense that the entire range of physical phenomena come to be systematised as immanent to the whole. This reduction of the whole of physical reality to a self-contained system by no means precludes the exposition of details in terms of transeunt causality.

(*b*) Now, although a monistic form may be given to the system of all physical reality, psychical reality remains essentially pluralistic, and cannot be formulated monistically. In a certain sense, physical reality exhibits the kind of causality that is termed transeunt and no physical causality is strictly immanent. This is because the ultimate constituents of matter—if there are ultimate constituents—have, so to speak, no insides. A physical event must always and can only be described as a changing or unchanging spatial relation of one thing to another,—the ultimate 'thing' having no inner 'states' which can be said to change or to remain unchanged. Hence, the immanency ascribed to the processes occurring within a mentally isolated material 'body,' is only immanency relative to processes occurring within other mentally isolated material 'bodies.' Nevertheless the conception of immanency cannot be eliminated in the formulation of physical laws; because the effects upon one body due to transeunt action from another are modifications of what *would be happening* within the body were no such transeunt causality in operation. Hence, the analysis of transeunt process always entails reference to immanent process; yet the converse (as it seems) does not universally hold; that is to say, it seems that purely immanent processes occur within the experiences of a single Experient (Psychical Continuant), though perhaps never within the happenings of a single Occupant (Physical Continuant).

§ 8. The more general problem in regard to transeunt and immanent causality relates to the modes in which the two forms operate in conjunction with one another. When any complete event is described in terms both of transeunt and of immanent causality, it would appear that, in transeunt causation, the cause-event and the effect-event are simultaneous; but that, in immanent causation, the cause-event always precedes the effect-event. This view is in direct contradiction to the prevailing view amongst philosophers who profess to attach scientific significance to the antithesis between the transeunt and the immanent. Illustrations in support of my contention will be found in the body of my work, where the temporal relations between cause and effect are discussed. Where cause precedes effect, as in immanent causality, I hold, in agreement with other philosophers, that there is no temporal gap between the two; they are strictly contiguous or as Dr Broad expresses it *adjoined*. Similarly, in transeunt causality, so far as spatial-relations between the two concerned continuants can be assigned, strict spatial contiguity goes along with temporal co-incidence. The above account must be understood to be preliminary and in a sense provisional; for, on further investigation, it will be seen that the simple principle that I have laid down must be partially modified.

§ 9. The views advanced in Part III on the problem of mutual interaction between 'mind' and 'body' may here be sketched in outline; and it should be said at once that I adopt the common-sense dualistic position and am, therefore, largely concerned with reconciling this position with the claim of science to have succeeded in formulating psychical and physical processes in general but precise terms. The common-sense view expressed briefly is as follows. Certain physical processes occur in accordance with purely physical laws and are unaffected by 'mind'; and similarly certain psychical processes occur in accordance with purely psychical laws and are unaffected by 'body.' Again, there are critical instants when a physical cause occasions a psychical effect which I shall term a *sensation*; and there are critical instants when a psychical cause which I shall term a *volition* occasions a physical effect. Of these last two cases, the former I shall refer to under the heading physico-psychical causality; the latter, under the heading psychico-physical causality. Since sensations (immediately occasioned by a physical cause) often engender psychical processes terminating in an act of volition which in its turn initiates a physical process; and since this latter sooner or later produces a physical consequent which, at a critical instant, occasions a sensation, the whole system of action and interaction assumes a cyclic form. In such cases, action initiated from either side is followed by reaction initiated from the other. But there is no reason to suppose that the cycle is in all cases completed. On the contrary, some stimuli which initiate modification of sensation are not followed by a consequent volition which initiates modification in the physical world; and some volitions which initiate modification in the physical world are not followed by a consequent stimulus which initiates modification of sensation. Action followed by reaction is probably the exception rather than the rule.

The cyclic processes may be roughly schematised as exhibiting, *alternately*, transeunt and immanent process. The Greek letters ϕ and ψ indicate respectively 'physical' and 'psychical' occurrences, and an arrow stands for 'causing' as also for 'preceding.' Thus:

$$(1) \quad \phi_a \rightarrow \psi_1 \rightarrow \psi_2 \rightarrow \phi_b,$$

$$(2) \quad \psi_a \rightarrow \phi_1 \rightarrow \phi_2 \rightarrow \psi_b.$$

Here the action $\phi_a \rightarrow \psi_1$ is followed by the reaction $\psi_2 \rightarrow \phi_b$, and the action $\psi_a \rightarrow \phi_1$ is followed by the reaction $\phi_2 \rightarrow \psi_b$. While, moreover, these actions and reactions illustrate *transeunt* causality, the intermediate processes $\psi_1 \rightarrow \psi_2$ and $\phi_1 \rightarrow \phi_2$ I shall speak of as *immanent*.

In case (2), the relation of the originative volition ψ_a to the terminal sensation ψ_b illustrates 'purpose.'

In case (1), the relation of the physical occurrence ϕ_a (which initiates the cycle) to the physical occurrence ϕ_b (which terminates the cycle) raises a general problem which is as yet without any unanimously accepted solution. This problem must be approached from a new side.

The problem next immediately before us is that of psycho-physiological parallelism. The term 'parallelism' is the well-known figurative equivalent for one-one correspondence or one-one correlation. But, unfortunately, it is used with further implications of meaning, two of which are in flat contradiction with one another. In philosophical usage, parallelism is generally understood to *deny* causal relation between the psychical and physiological correspondents; but, in Science, no such denial is implied (except of course by those scientists who reject causality altogether and substitute invariability). Now the grounds for maintaining parallelism in the philosophical sense have nothing whatever in common with those for maintaining parallelism in the scientific sense. In fact, at least as regards neural and sensational processes, most uninstructed persons accept scientific parallelism and would (if it occurred to them) deny philosophical parallelism. They would say that, inasmuch as variations in sensation correspond to variations in neurosis (as they are informed by competent scientists) the former variations are certainly caused by the latter.

§ 10. Here it is to be noted that the scientific assertion of correspondence is one-sided, whenever (as seems inevitable) the notion of causality is superimposed upon that of invariability. Impartial correspondence would assert that, just as the causal antecedents of a sense-stimulus—which occasions a modification of sense-experience—are purely physical, so the causal antecedents of a volition—which occasions a modification in the physical world—are purely psychical. Scientists, however, mostly appear to maintain that it is a mere illusion to suppose that the processes of desire or feeling and cognition or thought which terminate in a volition are causally operative. They maintain that the really operative causality resides in the neural process which, in accordance with the correspondence theory, accompanies the conative and cognitive experiences. In short, whenever the psychical processes $\psi_1, \psi_2, \psi_3, \ldots$ follow one another in a temporal and invariable order, this is so *because* the physical processes $\phi_1, \phi_2, \phi_3, \ldots$ follow one another in a temporal and invariable order. They, thus, tacitly maintain a one-sided operation of transeunt causality. They assert that the sequence $\phi_1 \rightarrow \phi_2 \rightarrow \phi_3$ constitutes the cause of the sequence $\psi_1 \rightarrow \psi_2 \rightarrow \psi_3$, and this assertion entails that the sequence $\psi_1 \rightarrow \psi_2 \rightarrow \psi_3$ never constitutes the cause of the sequence $\phi_1 \rightarrow \phi_2 \rightarrow \phi_3$. Adapting our previous schematisation to the present problem, the scientists' view would be indicated thus:

$$\begin{tikzcd}
\psi_1 \arrow[r] & \psi_2 \\
\phi_1 \arrow[r] \arrow[u] & \phi_2 \arrow[u]
\end{tikzcd}$$

in contrast with

$$\begin{tikzcd}
\psi_1 \arrow[r] \arrow[d] & \psi_2 \arrow[d] \\
\phi_1 \arrow[r] & \phi_2 ,
\end{tikzcd}$$

where the vertical arrows (in both cases) stand for transeunt causality.

Of course, if causality were excluded altogether, so that the vertical arrows stood merely for simultaneity and the horizontal arrows merely for sequence, then there would be no relevant distinction between the two alternative modes of representing the facts. Now, the view of alternate action and reaction is partially expressed by saying that, in some cases ϕ_1 and ϕ_2 respectively cause ψ_1 and ψ_2, while in other cases ψ_1 and ψ_2 respectively cause ϕ_1 and ϕ_2. That is to say in cases where ψ_1, ψ_2, etc. stands for a sequence of sensations then these are related to the sequence of neural processes ϕ_1, ϕ_2, etc. as effect to cause. But in cases where ψ_1, ψ_2, etc. stand for a course of conative and cognitive deliberation, then (if this course is accompanied by any discoverable physiological processes *corresponding* to the course of the psychical processes) ψ_1, ψ_2, etc. are related to ϕ_1, ϕ_2 as cause to effect.

In Part III a still bolder view is put forward: viz. that just as there are countless cases in which physical processes do not immediately occasion any psychical processes whatever, so there are cases in which psychical processes do not immediately occasion any physical process whatever. This view may be termed *impartial dualism.* Or—expressing the same view in metaphorical but familiar language—what is maintained is that man is a genuinely causal agent in reference to which his bodily organism serves directly and materials outside his organism indirectly as *instruments of his will.* On this view, a volition is immanently caused by such purely psychical processes as feeling, desire, knowledge and thought to which there are no neural or physiological correspondents.

§ 11. Before attempting to give direct evidence in support of the theory of impartial dualism, the scientific objections to this view must first be met. Physical Science claims that, in such a cycle as $\phi_a \rightarrow \psi_1 \rightarrow \psi_2 \ldots \rightarrow \psi_n \rightarrow \phi_b$ theoretically completed knowledge would be able, from the physical nature of ϕ_a, to infer the physical nature of ϕ_b, apart from any reference to the intermediate psychical occurrences ψ_1, ψ_2, ... ψ_n. The chain of events would assume the form $\phi_a \rightarrow \phi_1 \rightarrow \phi_2 \rightarrow \phi_n \rightarrow \phi_b$, where ϕ_1, ϕ_2, ... ϕ_n would represent assignable physiological processes occurring within a given bodily organism. Now the impartial dualist may fully admit this contention of the physical scientist and yet adhere to the view which attributes genuine causality to the 'mind.' For, the initial cause ϕ_a, which operates from without the particular organism, does not enable science to infer the terminal effect ϕ_b, without consideration of the *special* sequence ϕ_1, ϕ_2,... ϕ_n which varies according to the *special* nature of the organism. The form of response or reaction set up in one organism (expressed by ϕ_1, ϕ_2, ... ϕ_n) differs from that set up in another. These differences must be taken into consideration if the specific nature of the effect ϕ_b is to be inferred. We must causally account for the differences in the intra-organic processes as between one organism and another. This account will entail reference to the *past* history of the individual organism and of its ancestors. But what is the nature of the cause that stamps upon this or that organism its own special mode of organic response? This speciality of response can be predicted, by means of ascertained rules of uniformity framed in purely physical terms; but *why* are such or such physical antecedents invariably followed by such or such physical consequents? The character stamped upon each organism—by reference to which alone physical effects can be inferred from physical causes—may be the consequent of psychical processes, operating in such invariable modes as (theoretically at least) can be formulated in terms of physiological habits or trends or properties. The supremacy of physical law within the whole range of the physical is not hereby overthrown when mind is taken to be a genuinely efficient agent; for the notion of law may imply mere invariability, whereas that of an agent implies causality.

§ 12. A consideration of the different ways in which invariability and causality may be logically related gives rise to some questions of the greatest philosophical importance. In some cases, we have well-assured ground for asserting invariability, and from such assurance venture precariously to infer causality. In other cases, we have well-assured ground for asserting causality, and from such assurance venture precariously to infer invariability. The former type of case is that in which our main reliance is upon the accumulation of wide and varied instantial evidence; the latter, that in which our main reliance is upon the precision and accuracy with which we can analyse single instances. The distinction between these two types of logical procedure is, I believe, roughly illustrated in many regions of scientific enquiry. But I wish to maintain that this logical distinction can be applied as a ground of division between two departments of knowledge. By direct introspection, I feel assured that I can assign the cause of any one of my acts of will; but it is only with considerable doubt that I should venture to formulate rules in accordance with which I invariably act. In virtue of this assurance I maintain that, in willing, I am both free and determined: *determined*, because my volition is not uncaused; *free*, because the immediate causal determinants of my volition are within my own consciousness.

Causal determination of the will cannot be based on the ground of any observable uniformity of behaviour on the part of myself or of mankind in general or of animals. This is partly because no universally applicable rules of behaviour can be formulated; but, more obviously, because I do not know in what precise points the determining antecedents of one action agree with or differ from those of another. In order to formulate rules of behaviour or conduct, I must obtain accumulative evidence upon which a precarious generalisation may be inductively grounded; and, when all that is conceivably possible has been carried out by inductive procedure, my reliance rests ultimately upon the direct assurance of causal determinism yielded by introspection.

§ 13. The above analysis is open to the charge of extreme *naïveté*. But, before attacking my position on this or other grounds, I ask my readers to note that my account of the will differs in some important respects from those given by others. Many disputants on the subject of freedom of the will have put determinism and freedom in antithesis, whereas the true antithesis is between determinism and indeterminism. This latter antithesis was (I believe) first explicitly put forward by Dr G. Ward, who was still more explicitly followed by Pearse and W. James. Sidgwick declares that in immediate consciousness we are assured of freedom, but he goes on to maintain that the determinism that seems to be almost demonstrated by a sort of induction contradicts the freedom that is introspectively revealed. Again, many writers who reject determinism, interpret determinism as being materialistic:—a view which I absolutely disclaim. Again Mill and others reject freedom on the ground that it assumes the *effects* of volition to be known *a priori* without recourse to experience; whereas the freedom which I maintain entails rather direct knowledge of the immediate *causes* of volition. The knowledge of which I have direct assurance is a knowledge of the purely psychical phases such as desire and cognition of which I can become aware by retrospective or introspective attention; and these factors present themselves to me as cause of this or that volition. I am quite ignorant of the physiological processes which issue in an overt physical movement; and it is only after actual experience that I can foresee the more or less remote physical effects of any act of will, as is abundantly established by psychological enquiry. And again it is only by means of an extended experience that I can venture to generalise with respect to the volitions which will follow upon any recurrence of the same externally presented conditions, since the intensity of my desires and the determinateness of my cognitions are subject to alterations in the course of time.

One other frequent misrepresentation of the question under dispute must be mentioned. It is alleged against the determinist that he has falsely attributed to the will a kind of causality which is borrowed from the mechanical type of causation appropriate only to physical phenomena; whereas, in truth, as history proves, it is the type of causation exhibited in human volitions that has been borrowed and falsely applied to physical phenomena.

§ 14. Some justification is needed for my devoting so large a space to the detailed discussion of such psychological or metaphysical topics as freedom and determinism in a work professedly logical. My excuse is that the psychological, metaphysical and logical aspects of these problems have not been properly disentangled; and that it is only by bringing these aspects into close connection with one another that we shall succeed in getting to the root of the matter. Many empirical psychologists have explicitly put forward the view that, whether or not freedom, in some metaphysical sense, is to be attributed to the will, at any rate psychologists must work on the hypothesis of determinism. In this way, they preclude any discussion as to whether psychological determinism is or is not incompatible with metaphysical freedom. Or again: Kantians have tried to reconcile transcendental freedom with empirical determinism. But this attempt needs a preliminary discussion of the logical relation between freedom and determinism; and, moreover, attributes freedom to the transcendental ego and determinism to the empirical ego. Now, in a philosophical treatment of such scientific conceptions as those of substance and causality, there is no place for a transcendental ego or any species of *Ding an sich*. The freedom attributed by science to the will is empirical in just the same sense as that in which determinism is attributed. What causally determines any act of volition is a temporal event or process manifesting the character of the psychical agent, just as what causally determines a physical consequent is a temporal event or process manifesting the character of physical agents.

In order, then, to present a consistent and comprehensive view of the philosophical principles underlying scientific constructions and inferences, it is necessary to examine in what way such conceptions as cause and substance and such antitheses as transeunt and immanent causality are actually employed in science. The form in which these conceptions enter into psychical science fundamentally agrees with and also fundamentally differs from that in which they enter into physical science. Problems of parallelism and interaction could not be fruitfully discussed—even in a preliminary logical survey—without entering into controversial detail when attempting to apply the logical points at issue to the scientific analysis of psychical and physical facts.

CHAPTER I

FACT AND LAW

§ 1. ASSERTIONS about the universe of reality fall into two distinct classes which may be designated (1) assertions of fact and (2) assertions of law:—where the terms fact and law are restricted to the sense in which, taken together, they include experientially certifiable propositions and exclude formal propositions. Other terms approximately synonymous to 'fact' and 'law' are 'concrete' and 'abstract,' or again 'categorical' and 'hypothetical'; but these terms are used too loosely to bring out the antithesis which rests really upon the fundamental distinction and relation between substantive and adjective. Although according to our analysis every proposition is to be interpreted in terms of both substantive and adjective, we may assert provisionally that in the abstract proposition or assertion of law, the adjective is the more explicit or solely explicit factor, whereas, in the concrete proposition or assertion of fact, the substantive is the more explicit factor. Assertions of fact may be statements either of a single fact, i.e. about a single substantive, or of several single facts summarised in a proposition which shall have the same factual nature as the several propositions of which it is a summary. Or again, a concrete proposition may express not a conjunction but an alternation of single facts, and in this case it will be of the same nature as the assertions that constitute the several alternants, though less determinate than any one of them. In discussing the nature of a factual proposition then, we need only consider the proposition which expresses a single fact, without conjunction or alternation. The subject term of such a proposition, which denotes a pure substantive without adjectival characterisation, is best symbolised as *S*, and '*S* is *p*' will stand for a single assertion of fact where *p* is the adjective characterising the substantive *S*.

§ 2. The first difficulty about the proposition '*S* is *p*' relates to what we may call the referential problem: in other words, to what subject is the predicate *p* to be referred when we assert '*S* is *p*'? For, if the symbol *S* is non-significant—and, in default of any adjectival characterisation, it is difficult to see what significance it can have—then the proposition '*S* is *p*' cannot be intelligently distinguished from, say, the proposition '*T* is *p*' where *T* is equally non-significant with *S*. If we agree that '*S* is *p*' and '*T* is *p*' are *different* propositions, we may yet look beyond them for a common class to which both terms *S* and *T* belong. This common class is denoted by the wide term substantive used in its very general sense; hence, as a further interpretation of our formulae, the two propositions to be distinguished may be rendered in the forms '*This* substantive is *p*' and '*That* substantive is *p*.' The introduction of the terms 'this' and 'that' serves to show that substantives can be distinguished apart from, and independently of, any adjectival characterisation; so that, starting with 'this substantive' and 'that substantive' we may complete our predication by asserting of 'this' or of 'that' either the same or a different adjective. As I have stated elsewhere, I regard the principle of distinction which is independent of characterisation as ultimately based on the psychological fact of separateness of presentment of the manifestations of reality. The predesignation 'a certain' best indicates this separateness of presentment; and thus the more adequate formulation of the factual proposition runs: 'A certain given manifestation is *p*.' The introductory indefinite being preparatory to the referential definite, we pass from the predesignation 'a certain' to the definite 'this' or 'that.' This transition is possible psychologically so far as we can identify and discriminate the *positions*, temporal or spatial, at which manifestations are presented in separateness; and such identification or discrimination of position is, I maintain, psychologically prior to any subsequent relating in space or time, no less than to all forms of qualitative characterisation. The significance of the word 'given' in our formula is two-fold; in the first place, it indicates all that is meant by the word 'real'; and in the second place, it anticipates the *general* nature of the characterisation which completes the predication. For what is given, otherwise called the determinandum, is presented under a certain determinable, symbolisable by the capital letter P corresponding to the little letter p. The process of thought being the further determination of the relatively indeterminate, a further amendment of the formula will be: 'A certain given P is p.' Those logicians who wish to introduce identity into their analysis of the proposition may be partially gratified by this recurrence of the same letter in both subject and predicate[1]; but the fact that, ultimately, the subject term represents *indeterminately* what is represented *determinately* in the predicate term, does not preclude the referential problem of the singular categorical proposition; a problem which has been met by the unique employment of the phrase 'a certain' which is preparatory to the definite 'this' or 'that.' So much for the factual proposition.

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[1] See Part II, Chapter I, § 9.

§ 3. Passing to the consideration of the abstract proposition or assertion of law, this may be expressed purely in terms of characterising adjectives, in the form 'p determines q.' Here the word '*determine*' demands special consideration. In our account of the simple categorical statement of fact, we spoke of determination by thought, and to apply determination in this sense to our abstract proposition, we should have to combine the abstract assertion 'p determines q' with the concrete assertion 'a certain P is p,' these two propositions together *determining* us to assert 'this P is q.' According to this interpretation of the word 'determine,' the abstract proposition may be said to express an *anticipatory* determination for thought; for it must be conjoined with the concrete proposition in order to determine any further assertion.

Many logicians have been satisfied with this merely epistemic account of the relation of determination—a position which is tantamount to identifying the thought relation implication, with the causal relation in its widest sense, as indicated by Hume's phrase 'objective nexus.' Here we may note that Kant, deliberately opposing Hume, took the relation of implication to apply only to thought in general, and to be the typical form of judgment corresponding to the category of causality, the causal relation having validity in an objective sense. In this contention Kant undoubtedly aimed at distinguishing the subjective or epistemic from the objective or constitutive relation; but on this matter of the very first importance his view has been very variously interpreted. Of all the interpretations I shall adopt that in which the two conceptions of determination are most widely opposed. Before entering into the detailed analysis of this position, we must refer back to the epistemic distinction between experiential and formal certification. For example, an arithmetical formula, expressing relations between numerical adjectives, is one that can be formally certified apart from particular experiences. In contrast to this, any proposition which formulates a law of nature can only be certified ultimately by means of particular experiences. Now in Mill's use of the phrase 'empirical uniformity' there seems to me to be involved a fundamental confusion between the epistemic and the constitutive points of view which it is immediately necessary to remove. Epistemically understood Mill's phrase points to the ultimate data, namely observed instances, upon which the generalisation under consideration is based; and since he holds that all generalisations about natural phenomena are established on this same basis, there should be no distinction for him between empirical uniformities and causal laws. Mill nevertheless hints at an ontological distinction between these two kinds of uniformity where, for instance, he asserts that the method of agreement cannot prove causal laws; for if, as seems probable, in using this phrase he meant the emphasis to fall on the words 'causal law,' he must have had an ontological distinction in mind; it is only if the emphasis were upon the word 'prove' that a purely epistemic point arises. The same confusion is apparent in his view that the causal relation involves not only invariability but unconditionality. In my own view this qualification of Mill's represents the ontological distinction between a universal of fact and a universal of law. Thus taking the two determinate adjectives p and q under the respective determinables P and Q, the factual universal may be expressed in the form 'Every substantive PQ in the universe of reality is q if p'; while the assertion of law assumes the form 'Any substantive PQ in the universe of reality would be q if it were p.' These formulae represent fairly, I think, the distinction which Mill had in mind; for my first formula may be said to express a mere invariability in the association of q with p, while the second expresses the unconditional connection between q and p. Or, as I have said in p. 252, Chapter XIV, Part I, the universal of fact covers only the actual, whereas the universal of law extends beyond the actual into the range of the possible.

§ 4. Now the introduction of the word 'possible' here requires us to summarise briefly the main senses in which the word is used in common thought and in philosophy:

(*a*) The possible may be understood as equivalent to what is capable of being construed in thought; in this sense it is equivalent to the conceivable. Now the effort to construe in thought an entity which has been expressed in verbally intelligible form can be analysed into a step by step process such that the combination of characters and relations constructed up to a certain point may present to us some further character which our thought is *compelled* to assign to the construction. What then constitutes the impossibility of the proposed construction is the attempt to replace this further character, which we were compelled to predicate, by another character which is positively opponent to the former. It is this positive opponency between characters, therefore, which constitutes the genuine inconceivability upon which non-existence is to be maintained. In other words, the *impossibility* of some one mental construction is derivative from the *necessity* of a contrary or opponent mental construction. Let us take the most familiar example: the non-existence of a collection defined at the same time as *two plus three* and as *seven*, does not depend directly upon the impossibility of mentally conjoining these two numerical predications, but indirectly upon the necessity of conjoining the predication *two plus three* with the predication *five*, of which *seven* is a positive opponent or contrary. It is not a question of difficulty—amounting to an apparent impossibility—of making a thought construction in accordance with a verbal formula that constitutes inconceivability and gives the true test of non-reality; but rather the positive necessity of making some determinate construction opponent to the proposed construction.

(*b*) A second meaning of the word possible is quite easy to define; it relates merely to the limitations of knowledge: so that we say it is possible that such or such may be the case, meaning to express the quite simple fact that we are not, at the time, able to make a positive assertion concerning the truth or falsity of the proposed proposition. In this sense of the word possible, there is nothing in the nature of the proposition itself, apart from person and circumstance, which can determine its being possibly true or not, and for it I prefer to substitute the word problematic. A special case of this type of possibility arises when an individual has in his possession knowledge of various truths which he has not combined in thought, so as to elicit by mere thought process some further truth. In default of this thought process, the proposition expressing this further truth is not known, and is therefore possibly true and possibly false for him. All the complicated formulae of mathematics and logic come within this class for the ordinary man who has not taken occasion, or who is intellectually incapable, of developing such knowledge. This consideration leads to a third meaning of possibility.

(*c*) Propositions may be said to be possibly true or possibly false, in an explicitly referential sense; that is to say, possibility here is a feature not intrinsic to the proposition itself, but only when considered in reference to some other body of propositions taken to be true. Any proposition, then, whose falsity or truth cannot be formally deduced from a given body of propositions, may be said to be possibly true and possibly false referentially to this body.

(*d*) The further meanings of the word possible are connected with the notion of natural law and its antithesis to what we have called fact. The general form of a law, exhibiting the constitution of nature, has been expressed 'If any substantive were characterised as p it would be characterised as q.' This proposition expresses a relation between the characters p and q indicative of the nature of the world of reality. If any two characters x and y are *not* so related, then the conjunction of x with any opponent of y would be said to be a *possible* conjunction. When speaking of any fact or event as distinguished merely by spatio-temporal position from other facts or events, such terms as necessary or contingent cannot be applied. On the other hand, when we describe the event by an enumeration of certain adjectives or characteristics finite in number, and therefore non-exhaustive, the nomic distinction between the necessary and the contingent has significance relatively to such *description* of the fact, though not relatively to the fact. Thus the fact may be described as a pqr which is x. And so described it will be nomically necessary provided that any substantive characterised by pqr would be characterised by x; but it would be nomically contingent if anything characterised by pqr were not necessarily x. Now the nomic necessity—anything characterised by pqr would be characterised by x—implies the factual universal that 'everything that is pqr is actually x'; whereas the nomic contingency 'anything that is pqr is not necessarily x,' does not imply the factual particular that 'some things that are pqr are not x'; i.e. the affirmation of law, or nomic necessity, implies the factual universal; but the negation of law—i.e. the affirmation of nomic contingency—does not imply the factual particular. The logicians who reject the contrast that I maintain between law and fact, identify in effect nomic necessity with the universal of fact, and nomic contingency with the particular of fact. The conflict between these two views is apparent in the special case in which a factual universal expresses only a contingency; that is to say, when 'Every *pqr* is x' goes along with 'Any *pqr* might be not-x': e.g. the merely factual universal that 'Every day is followed by night' is compatible with the statement of contingency that 'Any day might be not followed by night.' Now the possibility of joining these two statements depends upon *day* being defined by a definitely limited conjunction of characters; for, if our definition exhausted all the characters, it would render the sequence of night inevitable, and we should be confronted with a universal of law. Expressing this symbolically:—An event described merely as a *pqr* that is x may represent a contingency; though such an event could theoretically always be more fully described as a *pqruvw* which is necessarily x. It may appear, since by an adequate description a contingency thus becomes a necessity, that the notion of nomic contingency has therefore no application. But, if we consider precisely why the conditions *uvw*, say, have to be added to the conditions *pqr*, in order that x may necessarily follow, it is because *pqr* does not nomically necessitate *uvw*, and therefore that the relation of *pqr* to *uvw* is nomically contingent. Thus the abstract question whether the character x of the given event is necessary or not is unanswerable, since it is seen to be contingent relatively to the incomplete description *pqr*; and necessary relatively to the complete description *pqruvw*. The philosophical justification of the principle under consideration requires the postulate that any character such as x manifested in a particular event is ontologically dependent upon an assignable—and therefore finitely enumerable—set of characters *pqruvw*.

§ 5. In further explication of the formal distinction between the assertion of law and the ordinary or factual universal, we require to define the class expressed by the phrase 'Anything that might be p'; for there are limitations to this class. Thus, if there is a law of nature that anything that may be x would necessarily be not-p, then a thing defined as in the class x would be excluded from the class of things that might be p, and this class includes, not anything whatever, but only such things as have been defined by a character not necessarily precluding p. Now a class defined in this way is very different from an ordinary or factual class; for we cannot take a given case and say whether it belongs to the class or not; all we know of a thing whose character is determined as x, say (where x corresponds to 'Anything that might be p'), is that this character x would not, under the realm of natural law, prohibit its being p. The particular case in question might, however, have other characteristics which *would* prohibit its being p: thus, if it be characterised as x and y, where the character x does not prohibit p, while the character y does prohibit its being p, and we take the completed definition xy of the class to which the thing belongs, it could not possibly be p; but if we take the incomplete definition, which includes only x and drops y, then we may assert of the thing that it might possibly be p. A concrete illustration will make this point clear:—Let x stand for a railway journey, and let us suppose further that in any actual railway journey the train travels with a brake. Now as far as the definition railway journey is concerned, there is nothing that prevents the train travelling without a brake, and therefore any instance might be one of a brakeless train. If we now take *y* to stand for the precaution which actually prevents the train being brakeless, the complete determination of the case would preclude it from being one of a brakeless train. The proposition to which our illustration leads may be put in the form: 'Any railway journey with a brakeless train would be liable to accident,' and the force of this proposition obviously extends over what must be called a wider range than the whole class of actual railway journeys; if we assume as a matter of fact that all railway journeys use a brake. This illustration suggests a wide class of cases which indicate human foresight or prudence; and in all such cases the distinction between the nomically necessary and the factual universal is quite apparent. If any action that might be characterised as such or such would produce undesirable consequences, and if human conduct is actually determined by knowledge of such consequences, then, as a matter of fact, those actions will never take place. The nature of the actual occurrences is defined, on the one hand, by the circumstances which would make such or such conduct disastrous, and on the other hand by the knowledge on the part of mankind, of this fact. If the occurrence be defined only by the circumstances, we can say of it that it *might* be such or such; but if, to complete the determination of the case, we add the knowledge of the consequences on the part of mankind, then this complete determination prohibits the possibility of its being characterised as such or such.

An example resembling that of the brakeless train, is 'Any person caught trespassing on this field will be prosecuted.' This proposition applies not only to the persons who have actually been caught trespassing and who have therefore been prosecuted, but to persons who have trespassed and have not been caught; for it is true of these latter persons, as much as of the former, that if they had been caught they would have been prosecuted. The application of the proposition, therefore, again extends to the possible, and is not restricted to the actually existent; though this illustration differs from the other inasmuch as there *are* cases of uncaught trespassers, whereas we supposed that no train travelled without a brake.

Another illustration of the same principle may be taken from the sphere of physics. Thus from such a formula as—'retardation varies as the coefficient of friction'—it follows that if the coefficient of friction were reduced to zero, the retardation would be zero. But in actual fact there is no instance in which the movement of one body over another does not entail friction, so that the above proposition applies over a range beyond actual fact. The point of importance, therefore, is that an assertion of this type may be scientifically established as true, while there may be no case presented in fact to which it is actually applicable. If propositions of this kind were interpreted as merely existential or factual, the actual non-existence of the class defined by the subject term would render it a matter of indifference whether one or any other predicate term were substituted. It follows that a merely factual or existential interpretation of this type of proposition is totally inadequate, and that to express its significance the proposition must be understood as applicable to the wide range of what is possible, as contrasted with the narrower range of what is actual.

§ 6. There is a subtle case in which the notion of the epistemically possible and the contingent—i.e. nomically possible—are combined, indicated by the term potential in one of its applications. Thus using the symbols employed above, we may know of a particular object that it is *pqr*, and that being such it may also be *x*; and further that if it were *uvw* as well as *pqr* it would necessarily be *x*. In this sense we may say that its being *x* is hypothetically necessary—a term which Mr Bradley uses to define the possible. Here the force of the word *hypothetical* is purely epistemic, and as thus applied it means that we do not know whether the thing is or will be *uvw*, knowing only that it is *pqr*. The term *necessary*, however, is used ontologically or nomically, and means that anything that is *pqruvw* would necessarily be *x*. Now I have to maintain that, given *pqr*, *x* cannot be said to be hypothetically necessary unless it is possible that anything that is *pqr* may also be *uvw*. We cannot therefore define the possible as equivalent to the hypothetically necessary, because the proposed character *x* is not even hypothetically necessary unless the junction of *pqr* with *uvw* is itself nomically possible. The meaning of the term potential, then, when the given thing, known to be *pqr*, is said to be potentially *x*, involves first epistemic possibility, i.e. we must not know that it *is uvw*; and secondly nomic possibility, i.e. anything that is *pqr may be uvw*. The most important use of the term potential coming under this wide head requires reference to a continuant subjected to transeunt causality. Thus to say of a solid strong body that it is potentially capable of resisting a certain measurable degree of pressure implies, from the epistemic point of view, that our known data do not include the knowledge that such a big pressure will be actually applied; that such a force may be applied is therefore hypothetical, or more precisely, epistemically possible. But further, to convey the full significance of potential resistance, this epistemic possibility must be combined with the negative fact that there is nothing in the laws of nature, and in particular in the character of the body itself, which would prevent this large force being applied. Shortly, then, the potential resistance of a body means epistemically that we do not know whether a certain force will be applied or not; and ontologically or nomically, that there is nothing in the nature of things to prohibit such force being applied.

CHAPTER II

THE CRITERIA OF PROBLEMATIC INDUCTION

§ 1. THAT induction is the inverse of deduction is a truism; but it is worth while to develop this truism in its various aspects. In its simplest form this inverse relation is exhibited by the change of place of premisses and conclusion, for roughly deductive inference consists in the passage from 'All *P*'s are *Q*' to 'Certain given *P*'s are *Q*'; and inductive inference in the passage from 'Certain given *P*'s are *Q*' to 'All *P*'s are *Q*.' Whereas this deductive inference is formally demonstrative, the inductive inference is obviously only problematic, and in general of a low degree of probability. The logical theory of induction may be developed by showing in what respects the degree of probability of such an inductive conclusion depends on the aggregate nature of the instances examined.

The general procedure in an inductive process is as follows: certain given instances are noticed as being characterised by certain adjectives—say *P* and *Q*—and we proceed to look for other instances characterised by *P*, in order to discover whether they are also characterised by *Q*. In the preliminary stages of induction, where *P* and *Q* jointly characterise certain observed instances, the sole factor which decides us to search for other instances of *P* in order to discover whether they are *Q*, rather than for other instances of *Q* to discover whether they are *P*, is that we have observed instances of *Q* which are not *P*, while so far every observed instance of *P* has been *Q*. In other words, we already know that 'Not all *Q*'s are *P*,' and therefore our enquiry is restricted to the question whether 'All *P*'s are *Q*.' The search for new instances to which we are thus prompted constitutes the preliminary process called *discovery*, and these instances are presented to us in one or other of two ways. They may either occur in the course of nature, and be discovered by active search in appropriate places and at appropriate times; or, on the other hand, we may have the means of producing them at places and in times where the course of nature, if uninterfered with, would not have exhibited such instances. These two kinds of active search are briefly denominated non-experimental and experimental: both imply activity prompted and guided by a definitive purpose.

§ 2. The use of experiment in discovery can only be accounted for by anticipating a discussion of what is to be understood by the somewhat vague term 'uniformity.' We speak of experiment as an interference with the course of nature; but we do not in any sense conceive that by such interference the laws or uniformities of nature are violated; for of the laws or uniformities of nature we may provisionally say that they do not prescribe the dates and places at which phenomena will occur, except so far as the dates and places at which other phenomena have occurred have been determined. If then the human will has the power of directly producing phenomena, the *course* of nature is modified, though the *uniformities* of nature are not infringed. The phrase 'uniformity of Nature' thus involves a certain ambiguity: it may mean uniformity in the *course* of nature independently of man's interference; as, for example, in the continued elliptic motion of the planets, or in the upward convection of heat through the atmosphere; or it may mean the aggregate of the laws or uniformities which are obeyed even when there has been human interference; as, for example, in the construction and working of Foucault's pendulum, or in carrying a hot body from one place to another. Whether, as in the first case, there is no interference, or as in the second case, human purpose intervenes, the laws of gravity and of convection of heat are equally unviolated; and in either case, the phenomena observed will afford means for studying the uniformities in accordance with which the operations of nature take place.

Further, from a certain point of view, uniformity in nature holds even when man interferes; for nature includes man, and we shall here assume that voluntary action obeys laws which as such are psychological, and exhibit the nature of man himself. Thus, if we suppose the occurrence of a definite purpose to be the immediate cause of the time, place, and manner of a certain interference in the course of nature, the formation of this purpose may be assumed to have depended upon antecedent psychological conditions, and thus to exhibit the kind of uniformity which is characteristic of man in his capacity of voluntary agent. The use of man's power to interfere with the course of physical nature is prompted not only by the purpose of acquiring further knowledge, but also by utilitarian ends. Thus the face of the physical world is, at the present time, totally different from what it would have been if the laws of physical nature alone had been in operation. We must therefore recognise a partial independence together with a mutual interaction between psychical and physical process, each following its own laws and also affecting the phenomena of the other. And this interdependence resolves the apparent paradox of scientific experiment, which consists in interfering with the *course* of nature, with the purpose of discovering more determinately the *laws* of nature.

§ 3. Returning now to the search for new instances; it may be assumed, in the simplest case, that all the instances of p so far examined have been discovered to be q; or, rather, more precisely, that none of them have been discovered to be other than q; for the character q may, in certain instances in which we have detected p, be beyond our power of observation. We thus arrive at an enumerative universal, 'All examined p's are q,' and this proposition constitutes the inductive premiss from which we venture to infer with a lower or higher degree of probability that 'All p's are q.' Now this summary or enumerative premiss may have very different degrees of value as evidence for the universal; we will therefore proceed to sketch in outline the different tests by which its value may be estimated.

In the first place, if this positive premiss stood alone it would in general have very little value; only when it is combined with one or more complementary propositions which taken together mutually support one another, has it serious evidential value. To constitute such a complementary, a proposition must have as its subject term a substantive with characters opposed to that of the subject of the positive premiss. This introduces the general notion of the determinable and its opposed determinates, and is explained in detail in the chapter on eduction, where I discuss the employment of intensional and extensional intermediaries. Here we will simply point out that, in calling a set of premisses complementary, we are extending or modifying the use of that term from the sense in which it denotes pairs of propositions like 'Every p is q' and 'Every non-p is non-q,' so as to cover an indefinite number of propositions corresponding to the indefinite number of values of P which have been correlated with an equal number of different values of Q. Thus the inductive premiss may be represented as a set of complementary enumeratives:

Every examined p is q
Every examined p' is q'
Every examined p'' is q''
....

and corresponding to these premisses, the inductive conclusion may be represented as a set of complementary universals:

Every p is q
Every p' is q'
Every p'' is q''
....

It must not be supposed that each several of these premisses constitutes by itself the evidence for the corresponding universal; on the contrary, the several premisses taken jointly constitute the experiential data upon which the strength of evidence for each of the several universals depends. This kind of compound induction then, which aims at discovering evidence that the value of Q depends upon the value of P, does so by finding that in all examined cases the value of Q has varied along with variations in the value of P, and that it has been found constant whenever P was constant. Thus the notion of dependence has two sides: (1) that the constancy of the one variable entails the constancy of the other; and (2) that the variation of the one variable entails a variation of the other. The reader will note that the collection of data in compound induction of this type roughly resembles Bacon's Table of Degrees and, somewhat less closely, Mill's Joint Method of Agreement and Difference.

§ 4. In rough or pre-scientific induction of the kind just described, it is not assumed that we are dealing with simplex variables, nor even with complex variables that have been analysed into their simplex factors: thus on further analysis we might afterwards discover that (say) $p = ab'c$, that $p' = a'b'c$, and that $p'' = ab''c'$; and similarly with q. This leads to the consideration of another criterion which affects the cogency of inductive inferences, viz., what will be called the criterion of specification. For example: in the inference from 'Every examined p is q' to 'Every p is q,' there is a liability to generalise too widely—a danger which is great in proportion to the indeterminateness of the subject character p; hence the more specifically p can be defined, the less hazardous will be our generalisation. The question arises: *How* specifically the determinate character p must be defined in order to limit this generalisation? Now the different cases which we have examined will all have agreed in *certain* characters, while, as regards *other* characters, some instances will have differed from others. We may therefore conceive of a certain conjunction of characters —say *abcd*—which characterise every examined instance, and by including all these characters in our definition of the subject term, we limit our generalisation within strictly logical bounds. In the case before us the conclusion will then assume the form 'Every *abcd* is *q*'; and this strict specification prevents us from inferring any wider generalisation such as 'Every *abc* is *q*,' or 'Every *bcd* is *q*,' or the still wider generalisation 'Every *bc* is *q*.' An elementary illustration will help to explain the force of this principle of specification. Common experience had afforded mankind in early times invariable evidence of unsupported bodies falling to the earth; if from this they had inferred that all unsupported bodies would fall to the earth, they would have neglected a character common to all the observed instances—namely that of proximity to the earth; their generalisation ought therefore to have been restricted to the statement that every unsupported body *in proximity to the earth* would fall. There are many phenomena which can be observed by man in a region of space limited in some such way as this; and hence the generalisations based upon such observations should be limited to the regions in which the character is manifested. Of course this does not mean that natural phenomena are dependent upon absolute spatial conditions, but only that there may be material bodies, occupying particular regions of space, upon which the phenomena depend. The same applies to periods of time: absolute dating in time does not affect natural phenomena; but there may be types of events occurring within certain periods of time, upon which other occurrences within those periods depend. Thus, under the general principle of evolution, the forms in which the uniformity of nature is manifested will be very different at different periods; it would therefore be invalid to infer, from the recorded evidence that nations throughout history have been either preparing for or actually engaged in war, that this will be the case in future: not because absolute time, any more than absolute space, is relevant for the uniformities of nature, but because the occurrences within a particular period of time causally affect other occurrences within that period; just as material bodies within a certain region of space causally affect other material bodies within that region.

The principle of specification can only be approximately realised in practice; for practically it demands that the instances examined shall agree with one another in no characters over and above those which are used to define the range of the generalisation. But if all the instances of *abcd* for example, agree in only one or two other characters, say *uv*, our generalisation, though it ought strictly to be limited to the narrower class *abcduv*, may perhaps be safely extended to the whole class *abcd*. The generalisation approximates to certainty in proportion as the additional characters common to the examined instances decrease in number. Thus the principle of specification, expressed in familiar language, demands that an assortment of instances designed to establish a generalisation, should be as varied as possible within the range defined by the characters comprised in the subject term. In this form it is seen to be practically equivalent to the principle underlying the method of agreement, which requires that the instances upon which a generalisation is based should exhibit together the maximum of difference or of variety.

§ 5. The principle of specification applies to a single enumerative such as 'Every examined *abcd* is *q*'; but we have seen that the condition for the highest degree of probability is that the generalisation in question should be supported by a set of complementary enumeratives, and we proceed to consider what relations should subsist between these several enumeratives. The requisite condition in this case is that, under the several different enumeratives, the instances examined should agree with one another as closely as possible in all characters other than those in which they are known to differ. Thus within the *same* enumerative, the instances should *differ* from one another as far as possible in the other characters; and under *different* enumeratives, they should *agree* with one another as far as possible in the other characters. This latter requisite approximates in practice to Mill's method of difference, application of which demands that the instances examined should agree as far as possible. We thus have two complementary criteria, the one requiring *variety*, and the other *similarity*. These principles express the common practice of the uninstructed mind; we can only justify them when we enter into the relation of probability to induction.

§ 6. So far we have not referred to the number of instances examined as a criterion affecting the strength of evidence. In point of fact mere number does not directly strengthen the instantial evidence; its importance depends upon variety; and number counts only because, by increasing the number of instances under the method of agreement, their variety is *probably* thereby increased. Corresponding to number, what is required for instances under the method of difference may be denominated proximity; not because mere proximity in time or space is important, but because instances which are either temporally or spatially close to one another will probably agree in many characters which it may be impossible for us to analyse. Thus the analogous criteria of number and proximity are both only inferior substitutes for analysis. When under agreement we cannot analyse sufficiently the characters of the instances to enable us to assert difference between them in many respects, we have to rely upon mere *number* of instances, which are presumed to secure a probable maximum of difference. Similarly under the method of difference, when the elaborate analysis required to enable us to assert agreement in many respects between the instances is impracticable, we have to rely upon mere *proximity* which is presumed to secure a probable maximum of agreement. The term proximity here is to be understood to include besides what would be literally called spatial or temporal proximity, also reference to the same agent whose conditions are varied from instance to instance.

§ 7. The fact that the criteria of number and proximity are mere inferior substitutes for the more direct criteria of variety and similarity, at once suggests that the evidential value of examined instances really depends upon the extent to which our analysis enables us to assert agreement or difference in the characters of the compared instances; and the probability of a generalisation therefore varies with the degree of precision with which we are able to define the characters of the instances examined. This consideration throws light upon Mill's problem 'why, in some cases, a single instance is sufficient for a complete induction, while in others, myriads of concurring instances, without a single exception known or presumed, go such a very little way towards establishing a universal proposition.' Speaking in terms of mere number, intensional number is of much higher value than extensional number; that is to say the number of characters in which instances are known to agree and differ is of much greater evidential importance than the actual number of instances examined. But again the mere number of characters analysed is not directly important in itself, any more than the mere number of instances examined: the characters counted should be strictly *independent* of one another, and this requirement is exactly parallel to that which demands that instances examined should *vary* with one another. Any character whose presence *is dependent upon* the conjunction of a given set of characters adds nothing to their evidential value. And, similarly, any instance which *agrees with* a given set of instances in all the respects in which these agree with one another adds nothing to *their* evidential value. Therefore whenever, in order to construct the intensional criterion for problematic induction, we count characters which are not known to be independent, we are relying upon the likelihood that a good many of them *are* independent. And when, to constitute the extensional criterion, we merely count instances which are not known to be pertinently different, we are relying upon the likelihood that a good many of them *are* pertinently different.

§ 8. The above parallel applies to instances and their characters under what may be in general called the method of agreement, where the generalisation refers to the characters in which all the instances agree. A similar parallel may be drawn for the instances and their characters under what we may call the Joint Method or Method of Complementaries. Thus the larger the number of instances which agree in certain characters *relatively* to the total number of instances observed, the higher is the probability that some of these characters are dependent upon one another. And the larger the number of characters in which certain instances agree *relatively* to the total number of characters analysed, the higher is the probability that some of these instances will agree in other characters besides those analysed.

§ 9. We have said that the probability of a generalisation varies with the degree of precision or determinateness with which we are able to define the characters of the instances examined; this determinateness reaches its highest point when instruments of measurement can be employed; and this accounts for the high probability generally attributed to generalisations formulated in terms of mathematical conceptions. Thus a further criterion of probability rests upon the possibility of applying quantitative considerations. It may be pointed out that this criterion carries the one which precedes it one step further in the direction of determinateness: the earlier dealt with the number of characters belonging to different determinables; the later deals with the number of determinate characters distinguishable under the same determinable.

§ 10. The last criterion to be mentioned virtually sums up and further organises all the preceding criteria. It may be said to rest upon the comprehensive complexity with which a law correlating the examined instances can be formulated. The generalisation of course consists in extending such a formula to unexamined instances; and if we have been able to define with comprehensive exactitude the kind of complexity exhibited in the instances compared, then the probability with which the formula may be extended to unexamined instances is commonly held to approach very nearly to certitude.

§ 11. In discussing the general nature of the method of problematic induction, the relations of agreement and difference are those which have figured throughout as the two forms of analytic comparison. From this point of view there can be no method of direct induction which might not be denominated by Mill's phrase, the Joint Method of Agreement and Difference. It may be useful, therefore, to draw up a two-dimensional scheme corresponding to the two relations of agreement and difference, which will enable us to figure this method in imagination. Instead of speaking of instances under the method of agreement or difference, we shall speak of instances under the relation of relevant agreement or of relevant difference. Instances which relevantly agree will be figured in a set of parallel columns; and instances which relevantly differ, in a set of parallel rows. It will be assumed that of the instances schematised, those which relevantly agree with one another will have irrelevantly differed from one another as much as possible; and those which relevantly differ from one another will have irrelevantly agreed with one another as much as possible.

p	p'	p''
$(abcd)\,u\ v\ w\ldots$	$(abcd)'\,u\ v\ w\ldots$	$(abcd)''\,u\ v\ w\ldots$
$(abcd)\,u\ v'\ w'\ldots$	$(abcd)'\,u\ v'\ w'\ldots$	$(abcd)''\,u\ v'\ w'\ldots$
$(abcd)\,u'\ v''\ w\ldots$	$(abcd)'\,u'\ v''\ w\ldots$	$(abcd)''\,u'\ v''\ w\ldots$
$(abcd)\,u''\ v\ w'\ldots$	$(abcd)'\,u''\ v\ w'\ldots$	$(abcd)''\,u''\ v\ w'\ldots$

In the above table any one of the columns contains instances which agree in the relevant characters $ABCD$, while they differ in some or other of the irrelevant characters UVW; the column is supposed to be indefinitely extended so as to represent a collection of instances presenting the largest possible variety as regards the irrelevant characters UVW. Any one of the rows contains instances each of which differs from all the others in the relevant characters, while they agree as far as possible in all the irrelevant characters. It will be noted that the relevant characters have been bracketed, and the dashes affixed to the entire bracket—a mode of symbolisation which is intended to denote that the observer, being unable to analyse the complex $ABCD$ into its simplex factors, may have been forced to regard the variations as pertaining to the complex as a whole. In comparing instances in the same row, therefore, the observer knows that there is *some* difference in the compared complexes, though he may not know to what special factor within the complex the difference attaches. If for the bracketed $ABCD$, a single letter—say M—were substituted, it would represent the observer's ignorance as to the nature of the factor he was varying—whether it was single, or if complex, which of the simplex factors, and how many of them were being varied. Similarly of instances in the same column, the dashes affixed to the bracket *abcd* denote that the *same* variation is observed throughout all the instances, although the observer may not know to what simplex factor or factors it attaches. The different columns each representing a set of instances in the relation of agreement constitute what I have called a set of enumeratives; and to represent symbolically the generalisation inferred from this aggregate of instances, we must imagine the columns extended on the same pattern to infinity. The several columns constitute a set of complementary enumeratives, and are each problematically extended into a set of complementary universals; the final generalisation, representing each value of P as depending upon the correlated value of $ABCD$, including all these minor universals.

§ 12. A scheme of this kind does not of course represent the detailed criteria used to estimate the degree of probability of different generalisations; it suffices, however, as a basis for criticising certain popular views on induction. The word 'hypothesis' is often loosely used in this connection: all inductions, it is said, are hypothetical; or again every induction is based upon hypothesis. These two separate assertions are not—as is sometimes supposed—equivalent. That every induction is hypothetical presumably means that inductive generalisations must be accepted with some reserve as regards their probability; in short, that induction does not ensure certitude. Instead of speaking of induction as hypothetical, therefore, I prefer to speak of it as being problematic; meaning by this that inductive generalisations cannot be affirmed with certitude, but only with a lower or higher degree of probability, depending upon the aggregate nature of the instances used to establish them. When on the other hand it is said that every induction is based upon hypothesis, 'hypothesis' means assumption; and the assumption referred to is some such proposition as that nature is uniform. Thus if it be held that the proposition that nature is uniform is not certainly true, but only probably true, then the degree of incertitude which attaches to the uniformity of nature must be attached to any induction whose validity depends upon the assumption of such uniformity. But again in this case I prefer to use the word problematic; for what is meant is that there attaches to the induction at least as low a degree of probability as has been attached to the proposition that nature is uniform.

A third meaning of the word 'hypothesis,' as it is used by Jevons, Sigwart and Bosanquet for example, when they assert that induction is based on hypothesis, requires separate discussion. In particular Jevons maintains that hypothesis is the first of the three stages in the completed inductive process, the second stage being called deduction, and the third verification. This use of the word hypothesis to denote the mere formulation of a generalisation which it is proposed to establish, is, in my opinion, totally unjustifiable. This so-called hypothesis or proposal constitutes the first stage in a process of which the third and final stage is called verification; in these two stages, therefore, reference is made to one and the same proposition, which is at first propounded as *to be* proved, and finally asserted *to have been* proved. In short the relation between the two stages is precisely that which obtains in Euclid between the enunciation of a theorem which prefaces a demonstration, and the termination of the proof which concludes with Q. E. D. But, if the word 'verification' is used here in its natural sense to imply that the conclusion of the inductive process is 'certified,' then it is meaningless to speak of induction as hypothetical in the sense of problematic. Jevons, however, maintains both these views: namely that completed induction ends with verification, and that induction involves an application of the theory of probability, thus rendering all generalisations problematic. If it is urged that I have taken the term verification too literally, and that all that is meant is that the generalisation is confirmed, and not actually verified, at this final stage, then the proper account of the process is simply that at the first stage a generalisation is accepted with a relatively low degree of probability, and at the last with a relatively high degree of probability; and there is no need to introduce the term 'hypothesis.'

My criticism of Jevons' account of induction extends, moreover, beyond his use of the notions of hypothesis and verification, to the stage of deduction which intervenes between them. According to his analysis, the proposition formulated in the first stage is taken as major premiss in a deductive process; the minor premiss being supplied by observation or experimentation. With these two premisses entertained in the mind as possible, a conclusion is drawn on purely deductive principles, referring in general to a single kind of case. Then, either with or without experimentation, we examine an instance of the type to which the deductive conclusion refers: and if, on comparison, accordance is found between the conclusion deductively reached and the observation of this specially contrived instance, the final stage called verification is attained. In a certain sense, then, this deductive procedure includes all the purely *mental* part of what Jevons represents to be a complete inductive process. Thus the relation between deduction and induction in his scheme may be represented:

(*Deduction.*) If P_1 and P_2 are true, then C would be true;

(*Induction.*) If C is true, then P_1 is true;

where P_1 and P_2 stand for the premisses and C for the conclusion. Now I wish to point out that the process of inference from P_1 and P_2 to C is in accordance with a demonstrative principle; but that inference from C to P_1 cannot be governed by a demonstrative principle; it follows, therefore, that Jevons' attempt to reduce the principle of induction to the principle of deduction is vain. The explanation of this blunder is to be found, I think, in a confusion between hypothetical inference and categorical inference. The deductive process to which Jevons refers is a mere hypothetical inference, which might be written: the truth of the premisses *would* imply the truth of the conclusion; whereas the inductive process is a categorical inference, and might be written: the truth of the premiss *does* imply the truth of the conclusion.

§ 13. Let us now give credit to Jevons for the truth which lies concealed in his theory. It has been expressed by Whewell and others in the principle that the sole test of an inductive generalisation is accordance with facts. This principle I hold to convey a truth, but only a partial truth; for in the first place it neglects the variable degree of probability to be attached to a generalisation based solely upon accordance with facts; and in the second place it neglects the variable degree of accordance which could be attributed to the relation of the generalisation to the facts. Combining these two neglected considerations, the principle of problematic induction may be restated in the following form:

The degree of probability to be attached to a generalisation based upon facts varies directly with the degree of accordance between the generalisation and the facts.

This maxim does not, of course, claim to be expressed with mathematical precision; the whole problem of the theory of induction is to define as precisely as possible what is meant by 'varying degrees of accordance.' Roughly, however, the main factors upon which such accordance depends are the number and variety of instances covered by the formula, and the determinateness with which the formula fits the facts.

1. With regard to the number of instances, the generalisation ranges over an infinite number of possible cases; hence the larger the number of observed facts found to conform with it, the higher its degree of accordance—on the score of mere number.

2. With regard to variety of instances, the generalisation claims to apply irrespective of circumstance; hence the wider the range of variety of circumstance in the instances observed, the higher will be the degree of accordance of the generalisation with the facts—on the score of variety.

3. With regard to determinateness, the degree of accordance is high in proportion as the generalisation

fits the facts closely and precisely. Thus, if a formula is comparatively indeterminate, then it cannot be said to accord closely with facts, even though it may cover a large range. For example, the generalisation that bodies falling to the ground move more and more rapidly as they descend may be confirmed by observing an actual increase of velocity, in which case a certain degree of accordance could be said to obtain between the formula and the facts. But if the formula asserts that for every second the rate of movement increases by approximately 32 feet per second, and by measuring the actual fall of bodies it is ascertained that the velocity of their descent does actually increase at this rate, the degree of accordance in this case may be said to be high—on the score of comparative determinateness.

CHAPTER III

DEPENDENCY AND INDEPENDENCY

§ 1. AGREEMENT and difference—the two principles upon which every method of direct induction ultimately depends—are notions which may be further expounded and more precisely defined by a logical analysis of the kind of proposition which directly expresses the data of observation. Such a proposition assumes the form: certain observed manifestations are characterised by the descriptive adjectives *mnpqr*, say. Now this form of proposition is—in two main respects—different from that with which we have been chiefly familiarised in logical teaching. In the first place, the familiar terms of quantity, such as 'all' or 'some' are omitted; and therefore one important aspect of induction is that it represents inference from a proposition concerning 'certain cases' to a conclusion about 'all cases'. In the second place, the proposition expressing the data of observation does not distinguish between those characters that define the subject term and those that define the predicate term: that is to say, it does not assume the familiar form 'Everything that is p is q.' Hence in passing from the proposition that directly expresses the data of observation to the proposition that expresses the conclusion inductively inferred, two kinds of transformation occur. The first transformation is from 'certain' to 'every,' and depends upon the condition of *variance* amongst the manifestations recorded. The second transformation is from a proposition containing no characterised subject term to a typical proposition which has a characterised subject term as well as a distinct characterising predicate term. This is equivalent to splitting up the conjunction of adjectives *mnpqr* into, say, *pqr* to constitute the characterising description of the subject term, and *mn* the characterising description of the predicate term—a separation which is rendered possible by distinguishing amongst the descriptive adjectives *mnpqr*, those which are independent of one another—viz. in our illustration *pqr*—from those which are inferred to be dependent upon the former—viz. *mn*. Only by combining these two transforming processes, therefore, can we infer from the inductive *premiss* 'Certain manifestations are *mnpqr*,' the inductive conclusion 'All manifestations that are *pqr* are *mn*'; and the two essential conditions required are (1) for the transformation from 'certain' to 'every,' variance of the observed manifestations; and (2) for the separation of the subject characters from the predicate characters, establishment of independence amongst the several subject characters.

Closer enquiry into the first stage of this transformation shows that it takes place before any separation of the subject from the predicate characters; or rather, to express the distinction more suggestively, of the determining from the determined characters. Instances, collected on the ground of manifesting certain characters in common, will always have manifested other characters differing from instance to instance. These inconstant characters have been omitted or eliminated in our summary description *mnpqr* of the data of observation;

but it is in virtue of these omitted characters that the observed manifestations can be said to have the aggregate nature of variancy. Thus, in this preliminary process, we conceive the characters which maintain their cohesion as forming a constant combination which is incapable of being destroyed by variations in other concomitant characters, the cohesion being the stronger, the greater the degree of this variation. This process corresponds to the principle of agreement, which has two aspects: namely, the elimination of varying characters, and the retention of a combination of constant characters.

The result of this first transformation may be stated in the form: there is *some* relation of dependence amongst the characters *mnpqr*; and the characters being regarded as a dependent conjunction, we are led to the second stage of our enquiry, viz. which of them are dependent upon the others? That is to say, we have next to discover amongst the characters in the constant conjunction, those which are *independent* of one another, and which therefore constitute the determining characters, by the conjunction of which the others are probably determined. Again we rely ultimately upon observation of instances, which, in order to lead to the separation of the characters *pqr* as independent of one another from the characters *m* and *n* as probably dependent upon them jointly, must have been of such a nature that wherever one alone of the characters *pqr* has varied, then *m* and *n* will have been found to vary; and wherever all the characters *pqr* were jointly constant, *m* and *n* were found to be constant.

§ 2. From this point onwards, our observations may be conducted under so-called 'experimental conditions'; the object of enquiry being to discover the specific values of P, Q, R which determine specific values of M, N. The special value of experiment lies in the completeness and accuracy with which it enables the experimenter to define the relations of agreement and difference subsisting between the determining and determined characters in each of the several instances observed. To secure the highest degree of accuracy and completeness, it is generally necessary to have recourse to *experiment* in the strict sense in which it implies that we have been able ourselves to contrive the instances observed. In such a case the experimenter knows beforehand which characters can be taken, for his purpose, as *independent* of one another—in the sense that he can vary these at will, with the assurance that the others will remain constant—and which characters are to be taken as dependent upon the former, in the sense that he is awaiting their manifestation in ignorance as to whether they will prove constant or varying, or varying to this or that degree. Thus he determines with accuracy the determinate values p, q, r before the result of the experiment is known, and then measures with equal accuracy the determinate values m and n; in this way ascertaining the precise effect which follows upon a precise cause, where previous to the experiment, both the cause and the effect were defined with comparative indeterminateness. But such exact experimentation presupposes the separation of the dependent from the independent factors; the dependent characters being those whose determinate values the experimenter wishes to learn as the *result* of the experiment; while the independent characters are those the determinate values of which he knows *before* experimenting. As has often been pointed out, the experimenter may of course be mistaken on these points; and while varying one factor, be unintentionally varying another, which is causally dependent upon it. This mistake would involve the assumption that certain factors were independent which were in reality dependent; but the mistake which I propose next to examine is the supposition, or rather inference, that certain factors are dependent, when they are really independent. A fallacy of this kind arises only where it is impossible for the scientist to contrive a variation in the characters manifested in nature in constantly cohering groups.

§ 3. To illustrate such an incorrect supposition, let us suppose a variation in b in two instances symbolised as $abcdq$, $ab'cdq'$. These concomitances might be analysed either in the form $bcd \sim aq$ and $b'cd \sim aq'$, or in the form $abcd \sim q$ and $ab'cd \sim q'$. In the first analysis a is taken to be dependent, and only bcd independent of one another; in the second analysis $abcd$ are taken all as independent of one another. The fallacy that we are considering is the assumption that the former is correct when, in truth, the latter is correct; that is, the factor a has been falsely supposed to be dependent, when in reality it is independent. This incorrect analysis would lead us to infer, (1) since the variation of b alone entails no variation in a, that for all instances $cd \sim a$; and (2) since the variation of b alone entails a variation of q, that for all instances $bcd \sim q$, $b'cd \sim q'$. But these two inferences could not have been made from the correct analysis; that is, we could not have inferred that q would follow from bcd for all values of A, or that q' would follow from $b'cd$ for all values of A; because, as far as the given instances alone are considered, the causal factors include the determinate value a along with the determinate values of bcd. Again the fact that A is independent of cd invalidates the inference $cd \sim a$, since, as a matter of fact, a might have had any value whatever concomitantly with cd. These inferences show that the employment of the figures of agreement and difference requires us to select beforehand those characters that can be properly regarded as independent of one another, as distinct from those which are dependent jointly upon them; for in both cases error occurs because the symbol a is placed on the side of the dependent factors, when it ought to have been placed among the independent. A more general form of exhibiting this same fallacy is to recognise independencies as being of certain numerical orders. Thus the false analysis $bcd \sim aq$ represents the independency to be of order 3, when the correct analysis $abcd \sim q$ shows it to be of order 4. Although it is always mistaken to assume an independency to be of a lower order than is actually the case, there is no objection to provisionally assuming it to be of a higher order than it actually is. In fact, the conclusions derived from the employment of the figure of agreement are those in which independencies provisionally assumed to be of order 4 say, are proved to be really of order 3, by showing that one of the supposed determining factors may vary without affecting the value of the determined factor, and may therefore be eliminated from the determining group. Another way of expressing the fallacy under consideration, therefore, is to say that an independency is represented as of a lower numerical order than is correct.

It must be noted that the order of an independency is not absolute, but relative to the effect factor whose variation is under consideration. Thus if we suppose a further complication added to our original instance, this would assume the form $a'bcd \sim pq'$ which, by comparison with $abcd \sim pq$, leads, in accordance with the figure of agreement, to the elimination of a as inoperative upon p. This inference is correct on the assumption that as regards the effect p, the factors $abcd$ constitute an independent group. In respect of the effect p alone therefore, the order of independence is 3, i.e. $bcd \sim p$; while, with respect to the effect q, the order of independence was 4, since we could not eliminate a as inoperative in determining q.

CHAPTER IV

EDUCTION

§ 1. THE term eduction is chosen to describe the kind of inference which Mill speaks of as from particulars to particulars. In place of Mill's phrase, I should substitute inference from instances to instances, and in using the technical term eduction, I wish to point out where I agree with, and where I differ from, Mill's view of the relation of deduction to induction. If we consider the singular instantial proposition '*s* is *p*,' it might stand first as a conclusion deduced from 'Every *m* is *p*' and '*s* is *m*'; or secondly as a premiss which together with '*s* is *m*' leads to the inductive inference 'Every *m* is *p*.' Now according to Mill, a more ultimate analysis of the deductive or syllogistic argument reveals it to be founded upon instances such as s_1, s_2, s_3, ... which, being *m*, are also *p*, so that the universal 'Every *m* is *p*' contributes nothing to the factual data upon which the syllogistic conclusion '*s* is *p*' is based; and he leads the reader to assume that the single conclusion '*s* is *p*' is established with the same force as the universal 'Every *m* is *p*' from the instances of s_1, s_2, s_3, ... that are *p*. At this point, however, I differ from Mill, and distinguish the type of inference which from 'certain *s*'s that are *m* are *p*' concludes that 'Every *m* is *p*,' from the type of inference which, from the same premisses, concludes that a further instance of *s* that is *m* is *p*. The former is called induction, and for the purpose of distinction, I give to the latter the name of eduction—this term indicating that the conclusion merely goes outside the instances which constitute the premiss, while the inductive conclusion extends to all instances of an assigned character. Mill's statement that the evidence for the conclusion 's is p' is the same as the evidence for the universal 'Every m is p' is somewhat hasty; for on the surface it would appear that from the same evidential data the single conclusion can be drawn with higher credibility than the universal. For the present, however, we will postpone the question of probability, and define more precisely the nature of the instantial premisses upon which both the eductive and inductive procedure are based.

§ 2. In this connection we have first to criticise Mill's use of the term 'particular' in his analysis of the process of 'inference from particulars to particulars'; for he appears to assert that the inference to 's is p' is based merely upon premisses such as s_1 is p, s_2 is p, s_3 is p, where s_1, s_2, s_3 might stand for anything whatever; much as if we argued, because fire is red, and poppies are red, and Mr Webb's tie is red, that therefore the British Constitution is red. In short he has neglected the essential question of the mediating conception, through which we pass from given instances that are p to some new instance. This mediating conception is precisely equivalent to the middle term of the syllogism, and I therefore here represent it by the symbol m; only when it is known that s is m, and that m is a character common to s_1, s_2, s_3, all of which are characterised by p, can we infer with any semblance of probability that the new instance s will also have the character p, which was the proposed predicate in our conclusion. Thus, for inferring the proposed conclusion 's is p,' the minimum of instantial data required includes three propositions; viz., 's_1 is m,' 's_1 is p,' and 's is m,' where m and p are of the nature of adjectives or universals, while the instances themselves are of the nature of substantives or particulars. A simple example of such a triad of premisses is as follows:

Mars is a solar planet.
The earth is a solar planet.
The earth is inhabited.
∴ Mars is inhabited.

Here the only known point of agreement between Mars and the earth is that they are both solar planets, and from this very slender relation of agreement we infer with the lowest degree of probability that Mars is inhabited, because we know the earth to be so. The probability of this conclusion is strengthened, the greater the number of characters in which Mars is found to agree with the earth; e.g. its being near the sun, and having atmosphere and vapour. It would be still further strengthened, if other solar planets besides the earth were known to be near the sun, to have atmosphere and vapour, and to be inhabited. The more complete process of eduction thus exemplified may be represented in the following scheme:

(1) s is-characterised-by p_1 and p_2 and $\ldots p_m$,
(2) p_1 and p_2 and $\ldots p_m$ characterise s_1 and s_2 and $\ldots s_n$,
(3) s_1 and s_2 and $\ldots s_n$ are-characterised-by p,
∴ s is-characterised-by p.

Thus, in eduction there are three summary premisses, containing (*a*) the summary term 'p_1 and $p_2 \ldots$ and p_m' which is adjectival; and (*b*) the summary term 's_1 and

$s_2 \ldots$ and s_n' which is substantival; besides the substantival term s and the adjectival term p, which occur in the conclusion. The mediating term 'p_1 and $p_2 \ldots$ and p_m' will be denominated the *intensional* middle term, and the mediating term 's_1 and $s_2 \ldots$ and s_n' the *extensional* middle term, s and p being respectively the *minor* and *major* terms. The premiss containing s will be called the *minor* premiss; that containing p, the *major* premiss; and that containing neither s nor p, the *mediating* or *middle* premiss. The eductive scheme may be conveniently represented in the form of a chain, showing how the subject and predicate of the conclusion are linked up through the two mediating terms, thus:

Minor premiss Major premiss

$$s\check{\chi}\ p_1, p_2 \ldots p_m\ \hat{\chi}\ s_1, s_2 \ldots s_n\ \check{\chi}p$$

Middle premiss

where $\hat{\chi}$ = characterises, and $\check{\chi}$ = is-characterised-by.

§ 3. Previous logicians have rather awkwardly contrasted inference by analogy with inference by induction—some regarding analogy as the basis of induction, and others taking induction to be the basis of analogy. In what sense these two terms are used is not clear, except that induction is understood to depend primarily upon the number of instances known to be characterised by a certain adjective; while the force of analogy depends upon the number of adjectives that are known to characterise a certain instance. But it is essential to insist that neither by accumulating instances alone, nor by accumulating adjectives alone, can any inference be drawn, and that inference of this type, by whatever name it may be called, is governed by principles which underlie both induction and analogy—requiring an intensional as well as an extensional link. For example, no mere accumulation of instances $s_1, s_2, s_3, \ldots s_n$ that are p could give any probability that a new instance s will be p, unless s were known to have at least one character predicable of all these instances. And conversely, no accumulation of characters $p_1, p_2, \ldots p_m$ that are predicable of s, could give any probability that a new character p is predicable of s, unless p were known to be predicable of at least one instance having all these characters.

For the purposes of developing the principles of eduction in their relations to probability, a fundamental distinction must be made according to whether

(*a*) all the evidential data are in favour of s being p,
or (*b*) some of the evidential data are in favour, and others unfavourable.

In case (*a*) the eductive process leads to an inductive inference whose conclusion is universal; in case (*b*) to an inductive inference whose conclusion is class-fractional. The remainder of this chapter will be limited to case (*a*)[1].

Now the data favourable to the proposal s is p, fall into four heads:

(1) Intermediaries $p_1 \ldots, s_1 \ldots$, such that
$$s\breve{\chi}p_1\hat{\chi}s_1\breve{\chi}p$$
(2) Intermediaries $q_1 \ldots, t_1 \ldots$, such that
$$s\bar{\breve{\chi}}q_1\hat{\chi}t_1\bar{\hat{\chi}}p.$$
(3) Intermediaries $x_1 \ldots, u_1 \ldots$, such that
$$s\breve{\chi}x_1\bar{\hat{\chi}}u_1\bar{\bar{\breve{\chi}}}p.$$
(4) Intermediaries $y_1 \ldots, v_1 \ldots$, such that
$$s\bar{\hat{\chi}}y_1\bar{\hat{\chi}}v_1\breve{\chi}p.$$

These four cases will be recognised as all favouring the proposal s is p, because they constitute the different ways in which an even number of non-characterising links enter.

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[1] Case (*b*) will be treated in the Appendix to Part III.

§ 4. If now all the evidential items are of one or other of these four kinds, then they all co-operate in strengthening the probability that s is p. Speaking generally, the larger the number of data of this kind that have been established, the higher is the probability that s is p. But certain conditions must be fulfilled in order that any apparently new evidence should actually strengthen the required probability. First as regards the extensional aspect of the evidence. The enumerated set of instances $s_1 \dots s_n$ will count as n separate data, provided that every one of them such as s_n is uncharacterised by some of the adjectives that characterise all the remainder, i.e.

$$s_n \bar{\breve{\chi}} q \hat{\chi} s_1 s_2 \dots s_{n-1}.$$

In other words, the instance s_n counts as one additional item of evidence provided that it increases the variety of the evidence; which it would fail to do unless it had, besides a certain nucleus of characters common to all the other instances, some character opposed to the common characters of the others. Secondly, as regards the intensional aspect of the evidence: The enumerated set of adjectives $p_1, p_2 \dots p_m$ will count as m separate data, provided that every one of them such as p_m, does not characterise all the instances that are characterised by the remainder, i.e.

$$p_m \bar{\breve{\chi}} t \breve{\chi} p_1 p_2 \dots p_{m-1}.$$

Hence an instance s_n adds to the weight of evidence when it constitutes a variation upon the other instances; and an adjective p_m adds to the weight of the evidence when it is independent of the other characters.

§ 5. We are thus enabled to establish certain principles regulating the strengthening force of evidential data.

In inferring from examined subjects that a given subject has a property characterising these, we rely upon the likeness of the new subject to those adduced, and the force of any such new instance varies with the degree of resemblance to the adduced subjects, and with the degree of unlikeness amongst the examined subjects themselves. The more remotely the latter differ from one another, the stronger is the evidence that they will agree with the new subject in further points, beyond those in which they are known to agree. In other words, the more varied or non-congruent with one another the accumulated subjects, the stronger the evidence in favour of a certain congruence. Similarly with regard to the intensional aspect of the evidence; the predicates $p_1 \dots p_m$ which serve as intermediaries must be as independent of one another as possible, when used as evidence for establishing the dependence of a further proposed predicate upon these given predicates taken in conjunction. Summing up, then, for establishing a proposed congruence, the condition required is non-congruence amongst the examined subjects; and for establishing a proposed dependence, the condition required is non-dependence amongst the examined predicates.

These formulae represent the final inferred inductive conclusion in its two-fold universality; such universality being, on both sides, unlimited. In contrast to this unlimited universality, the evidence for such an inductive conclusion has to be exhibited in terms of what by observation and examination is *known*. Thus when all the subjects known to be characterised by $p_1 \dots p_m$ are also characterised by p, we infer inductively that all subjects so characterised, will be characterised by p.

And when again all the predicates that are known to characterise $s_1 \dots s_n$ also characterise s, we infer inductively that all the predicates which characterise this set will also characterise s. What we have here expressed in the form of a set of subjects generating a subject group, and a set of predicates generating a predicate group, is in effect equivalent to what was above explained as the intensional and extensional aspect of the eductive process.

§ 6. It will be observed that the instantial evidence for the proposition 's is p' does not point to any one given s or p, but to any subject characterised by the conjunction of predicates $p_1 \dots p_m$, and any predicate characterising the conjunction of subjects $s_1 \dots s_n$. We can therefore eliminate the explicit symbols s and p, and consider only what we have called the intermediary premisses, summed up in the proposition $s_1 \dots s_n$ are characterised by $p_1 \dots p_m$. This proposition will be called the summarised evidential datum, pointing to the conclusion that any unassigned subject characterised by $p_1 \dots p_m$ will be characterised by any unassigned predicate that characterises $s_1 \dots s_n$. This summary evidence contains mn atomic data, each additional subject and each additional predicate in the two conjunctions counting as one.

We now proceed to consider the precise condition required in order that each of these subjects and each of these predicates shall count as one, in the estimate of the evidence before us. The condition that the predicate p_m, say, shall count as an additional item, is that there shall be some subject, say σ_n, such that

$$p_m \,\bar{\chi}\, \sigma_n \,\breve{\chi}\, p_1 p_2 \cdots p_{m-1},$$

where there is also a subject σ_{n-1} such that

$$p_{m-1} \bar{\breve{\chi}} \sigma_{n-1} \breve{\chi} p_1 p_2 \cdots p_{m-2}$$

and so on. In other words, each additional predicate, p_m, p_{m-1}, etc., must be known to be independent of the conjunction of the remaining predicates. In this case the set of predicates $p_1 \ldots p_m$ will be called an independency.

A corresponding condition is required for the set of subjects $s_1 \ldots s_n$; i.e. there must be a predicate π_m such that

$$s_n \bar{\breve{\chi}} \pi_m \hat{\chi} s_1 s_2 \cdots s_{n-1}$$

as also a predicate π_{m-1} such that

$$s_{n-1} \bar{\breve{\chi}} \pi_{m-1} \hat{\chi} s_1 s_2 \cdots s_{m-2}$$

and so on. In other words, each additional subject s_n, s_{n-1}, etc., must differ in character in at least one predicate from the remaining subjects. When the set of subjects satisfies this condition, we shall speak of it as a variancy.

§ 7. Thus the parallel terms independency and variancy—the first applying to a collection of predicates, and the second to a collection of subjects—can be defined absolutely; i.e. without reference to any other named predicate or subject. Now such an independency must be distinguished from another set of predicates which are invariably found to be concomitant with the independent set in our instantial evidence. If p typifies such a concomitant predicate, p will differ from any of the predicates in the independent set in the point that there is no subject, such as σ, which is known to be characterised by the set $p_1 \ldots p_m$, which is known not to be characterised by p. Again the set of subjects $s_1 \ldots s_n$, which constitutes a variancy, must be distinguished from another set of subjects which are found to be congruent with the set $s_1 \dots s_n$ in all their known common characters. If s typifies such a congruent subject, s will differ from any of the subjects in the variant set in the point that there is no predicate such as π which is known to characterise the set $s_1 \dots s_n$, which is known not to characterise s. There are thus two complementary aspects of our evidence, which have been summed up in the negative form (1) that no subject known to be characterised by $p_1 \dots p_m$ is known not to be characterised by p; and (2) that no predicate known to characterise $s_1 \dots s_n$ is known not to characterise s. These two negatively formulated conditions may be otherwise expressed affirmatively; viz. first that every subject that is known to be characterised by $p_1 \dots p_m$ is also characterised by p; and secondly that every predicate that is known to characterise $s_1 \dots s_n$, also characterises s. These conditions in our ascertained knowledge are equivalent to the statement that all our evidence is unexceptionally favourable towards the conclusion s is p; i.e. the evidence comes under our first heading (*a*). Where s is unexceptionally congruent with the collection $s_1 \dots s_n$ as far as our knowledge reaches, we are led to the inference that it will be congruent with this set in all other unknown as well as known predicates. Again where p is unexceptionally concomitant with the set $p_1 \dots p_m$ as far as our knowledge reaches, we are led to the inference that it will be concomitant with this set in all other unknown as well as known subjects.

§ 8. Having brought out the possible parallels between substantives and adjectives, or subjects and predicates, as far as analogy permits, we have arrived at a point where an irreducible contrast between these two categories prevents the further formulation of the intensive and extensive aspects of eduction on precisely the same lines. The substantive or subject of the simple proposition 's is p' is ultimately identified and distinguished from the adjective or predicate by a definite type of act of thought. The subject s is definitive and nameable, as well as identifiable and distinguishable by such extrinsic relations as temporal and spatial position; such identification may therefore be said to be determined by an act of separation. On the other hand the predicate p stands for an adjective or character predicable of different substantives by means of an act of comparison or discrimination; predication, therefore, may be said to be determined by an act of discrimination.

§ 9. My whole philosophical attitude depends upon the recognition of a fundamental distinction between these two types of acts, specially denominated separation and discrimination—a distinction which corresponds in my general view to that between the particular and the universal. Inasmuch as substantival separation is a necessary preparatory step to identification or comparison of character, separation may be said to be prior to discrimination; and this entails a further important contrast between subject and predicate. Thus there is, amongst substantives, nothing corresponding to the distinction and relation which obtains amongst adjectives between determinables and determinates; substantives proper, i.e. existents, are necessarily *distinct* just because they occupy positions in the *same* totality of time and space; whereas determinates which are *opposed*, are those which belong to the same determinable.

CHAPTER V

PLURALITY OF CAUSES AND OF EFFECTS

§ 1. It has become common in modern logical works to deny the applicability of the plurality of causes where scientific analysis has succeeded in the final formulation of natural laws. While agreeing in the main with this logical position, I consider that the notion of plurality of causes and effects is applicable where such scientific analysis is incomplete, and that its discussion therefore has a place in the logical foundations of science. Since there are various senses in which the phrase is used, it will be convenient to arrange the discussion under a number of heads of discourse.

The most elementary notion of plurality of causes is that which Aristotle called 'the fallacy of the consequent,' meaning by this what now-a-days we call the fallacy of simply converting a universal affirmative. Thus, to infer from 'Every c is e' that 'Every e is c,' or from 'If c then e' that 'If e then c' is what in modern logic we designate as the confusion between a proposition and its complementary; and this fallacy has practical application where c_1 (say) stands for the characterisation of a cause, and e_1 for the characterisation of its effect. It is obviously fallacious to infer from the manifestation of the character e_1 that the character c_1 will have been manifested; for the effect e_1 may have been due in any given instance to some other cause, say c_2, and this possibility entitles us to speak familiarly of plurality of causes in cases where simple conversion is illegitimate. This first application of the conception may be expressed either as an *alternative* of predications, or as a *conjunction* of propositions. Thus, starting with the effect e_1, the proposition expressing plurality assumes the alternative form; 'Every e_1 is either c_1 or c_2 or c_3, etc.'; but starting with the cause, it assumes the form of a conjunction of propositions, namely; 'Every c_1 is e_1, *and* every c_2 is e_1, *and* every c_3 is e_1, etc.' It is usual in the general exposition of plurality of causes to employ the alternative form of the proposition; and this has led to the view that it is impossible to infer from the characterisation of an effect what in any instance may have been its specific cause; for the alternative form of proposition 'Every e_1 is either c_1 or c_2 or c_3, etc.' renders it impossible when the effect is characterised merely by the character e_1, to determine amongst the several alternatives c_1, c_2, c_3, etc., which has actually operated in any given case. But there is nothing in the fact of such plurality of causes as this incompatible with the view that a sufficiently precise characterisation of the effect enables us to assign the specific cause in any given instance. For example, the alternative proposition can be consistently held along with the determinate propositions 'Every e_1f_1 is c_1,' and 'Every e_1f_2 is c_2,' and 'Every e_1f_3 is c_3,' etc.; and it is important to note that this holds equally of the sufficiently precise characterisation of the cause. It is a fundamental error common to most accounts of plurality to suppose that something is true of the relation of effect to cause, which is untrue, *mutatis mutandis*, of the relation of cause to effect. There is absolute reciprocity between cause and effect, and insufficient determinateness in the assignment of either prohibits inference, whether from effect to cause, or from cause to effect. The apparent want of reciprocity is simply due to an imperfection of terminology. The word cause is understood to denote a *completed* assignment of cause-circumstances; while the term effect is used to denote an *incomplete* assignment of effect-circumstances. An incomplete assignment of cause does not enable us to infer a determinate effect, and hence, in this sense, it is not true that the same cause involves the same effect. On the other hand, a completed assignment of the effect does enable us to infer a determinate cause; and hence the statement that the same effect does not involve the same cause is equally false.

§ 2. The question which naturally next arises is what constitutes a complete assignment of cause or of effect, such that we are able to infer from one to the other? There is, in my view, no general answer to this question: each case must be treated *ad hoc*. Complete assignment of cause will be relative to the more or less arbitrary assignment of the effect-character; and where effects are assigned with perhaps equal degrees of indeterminateness, very different degrees of determinateness might have to be assigned to the cause in order to permit of inference. This holds equally with regard to the complete assignment of an effect. It will help to elucidate the problem to distinguish under the term 'cause,' the 'completed cause' from any 'cause-factor'; and, under the term 'effect,' the 'completed effect' from any 'effect-factor.' Thus, relatively to any effect-character *e*, we shall speak of the conjunction *abc* as constituting the completed cause, when the universal proposition 'Every *abc* is *e*' holds. In this case, the several characters *a*, *b*, *c*, are cause-factors of the given effect characterised as *e*; and the truth in the doctrine of plurality of effects is expressed in the statement that a recurrence of *a* alone or of *b* alone or of *c* alone does not entail a recurrence of any the same effect-character such as *e*; in other words, the mere recurrence of a cause-factor does not ensure the recurrence of any the same effect-character. When then we speak of a completed assignment of the cause, we mean simply such an assignment as will ensure identity, in its several recurrent manifestations, of some character in the effect. Similarly in the case of effect: here we start with some cause-character *a*, and proceed to establish some conjunction of effect-characters, say *pqr*, such that wherever the conjunction *pqr* is manifested, we can infer the causal operation of *a*; the conjunction *pqr* is then denominated the 'completed effect' of *a*. On the other hand, the several characters *p*, *q*, *r*, are effect-factors from neither of which alone could the operation of *a* have been inferred. This last fact expresses the truth inherent in the doctrine of plurality of causes. From the effect-factor *p* alone, or *q* alone, or *r* alone, we could not have inferred any such determinate causal factor as *a*; while from the conjunction of these characters, which we have called the completed effect, the character *a* of the cause may be safely inferred.

§ 3. To bring out the fundamental principle that when any invariability of relation can be established, it is always a determinate conjunction of characters from which some other character can be inferred, we will symbolise the completed cause of *p* as *abcd* where

'Every $abcd$ is p,' and the completed effect of a as pqr where 'Every pqr is a.' We do not expect in general to be able simply to convert these uniformities in the forms 'Every p is $abcd$' or 'Every a is pqr'; nor, from the universal 'Every $abcd$ is p' do we expect to be able to infer that every p is a, or that every p is b; or from the universal 'Every pqr is a' that every a is p, or every a is q, etc.; for any given case of p might have been $a'b'cd$, and any case of a might have been $p'q'r$. So from any one effect-character p we should not in general be able to infer any one of the cause-factors which together constitute its completed cause; and from any one cause-character a, we should not in general be able to infer any one of the effect-characters which together constitute its completed effect.

For brevity's sake we will here replace the compound symbol $abcd$ by the letter x, and the compound symbol pqr by the letter z. We then have the two universal propositions 'Every x is p' (where p is an effect-character, and x its completed cause), and 'Every z is a' (where a is a cause-factor, and z its completed effect). As we have already pointed out, the doctrine of plurality may be expressed in two ways—either in the alternative form of predication, or in the conjunctive form; and so the above discussion may be summarised either in the alternative form 'Every p is x, or x', or x'', etc.' and 'Every a is z, or z', or z'', etc.' or in the conjunctive form 'Every x is p, and every x' is p, and every x'' is p, etc.' and 'Every z is a, and every z' is a, and every z'' is a, etc.' Colloquially expressed these propositions are embodied in the statements that 'in different instances *different* causes point to the same effect'; and that 'in different instances *different* effects point to the same cause.' Our immediate problem, then, is to examine further into the significance of the term 'different,' and to enquire into its adequacy for expressing the true meaning of plurality.

§ 4. When we assert that under different antecedent conditions the same consequent is manifested, we may be understood to be simply asserting that any such proposition as 'Every x is p' means that p can be predicated in all cases—differing in an indefinite number of characteristics from instance to instance—where the character x is manifested. But the mere difference of circumstances in such cases, being followed by the same effect, cannot be said to constitute plurality of causes; for, if a character, expressed by the determinable V say, varies quite indefinitely without affecting the character p, then the various values of V which may be manifested from instance to instance are simply eliminated in the proposition that 'Every x is p.' In order to speak properly of plurality, the case must be such that *some* but not *all* of the possible values of a determinable can be substituted in the universal proposition; for instance where, X being the determinable, 'Every x is p,' 'Every x' is p,' 'Every x'' is p,' are the only three values of X which yield p. The conception of plurality, therefore, requires us to refer to a restricted range of alternatives; some, but not all, values of X point to p; some, but not all, values of Z point to a. Further x, x', x'', it will be observed, are contraries, opponents or disjuncts, and so the term 'different' in this account has an added significance. A rough example will illustrate this point: death is sometimes caused by poison, sometimes by a blow on the head; the mere *difference* between these two causes does not bring out the full significance of plurality, for poison without the blow, or the blow without poison, would produce the same effect death, and these are contrary circumstances. A similarly rough illustration of plurality of effects might be taken from the fact that both a picture and a pattern point, as effects, to human action as cause; here again the full significance of plurality is brought out not merely by the difference between pattern and picture, but by the contrariety of these effects, one being a picture which is not a pattern, and the other a pattern which is not a picture; while both opposed effects point to the same cause, namely human purpose or agency. In both these rough illustrations where contrariety or opponency are substituted for mere difference, we must emphasise the further consideration that the range of opponency which gives significance to the idea of plurality is restricted, and is therefore to be contrasted with a completely unrestricted range where possible variable values would be simply eliminated. It is not every splash of colour that points to human purpose, but, in our assumed example, only those which are pictorial or symmetrical; and it is not every condition of the head or inner organs that would point to death, but only some selected and definable conditions.

§ 5. As stated in the previous section, each of a number of opponent cause-characters x, x', x'' (say) entails the same effect-character p, when conjoined with other cause-characters respectively, y, y', y'' ... (say), X and Y being determinables indefinitely variable. But it must also be recognised that each of a certain *finite* set of values x_1 x_2 x_3 (say) of X, when conjoined with the *same* value

y of Y may entail the same value p of the effect-character P. This fact is best illustrated by a graph in which the abscissa represents the variable cause-character, and the ordinate the variable effect-factor, and where—other relevant circumstances being unchanged—a horizontal line meets the graph in two or more points A, B, C, D.... If one observation is represented at one point A, there is a chance—which however is small—that the second observation should be on another point B where the horizontal through A meets the curve; but this initially small probability decreases continuously with every fresh instance. This familiar fact requires us to modify somewhat our formulation of the figures of Demonstrative Induction. Thus, as regards the figure of Agreement, it was said that if any two values, say a_1 and a_2, of the cause-factor A (all other factors, b, c, d, e remaining constant) entail the same effect value p, then P is independent of A (under the circumstances $bcde$). But such independence would be falsely inferred if a_1 and a_2 were two values which yielded the same effect p. The probability, under the ordinary circumstances of experiment and observation, of precisely these two values occurring is in general so small as to be negligible; and in order to diminish even this small probability more than two values of A should be experimentally instanced. A similar correction is required for the figure of Difference. Thus, if d and d' have yielded different values p and p' of P (under otherwise constant circumstances) we cannot infer that literally *any* other value of D would yield a different value of P; for there is a small chance that in another instance we might happen to hit upon one of the values of D that yields the same value p of P as that yielded by d.

§ 6. We have attempted to prove that the relation between cause and effect is reciprocal in the general sense that whatever is true of cause to effect, will be true of effect to cause, whether the relation asserted is in the form of a universal, a particular, a conjunctive, or an alternative proposition. We now proceed to investigate how the relation of cause to effect can be rendered reciprocal in the more precise sense in which it involves the conjunction of the two complementary universal propositions 'Every C is E' and 'Every E is C.' Now in formulating any universal proposition, we begin by taking as predicate term an arbitrarily assigned character which may stand either for a cause-character or for an effect-character. We then attempt to find some conjunction of characteristics from the manifestation of which the arbitrarily assigned character may be inferred. This conjunction of characteristics is denominated 'the completed cause' when the original character stands for an effect; and is denominated the 'completed effect' when the original character stands for a cause. Thus, in general, starting with the predicate term p, we attempt to establish such a proposition as 'Every *abcd* is p.' If, at this point, we turn from p to *abcd*, and attempt to find a universal mark from which the conjunct character *abcd* could be inferred, we discover a mark definable by some such conjunction as *pqr*. We should then have the two universals 'Every *abcd* is p' and 'Every *pqr* is *abcd*,' where *abcd* defines the completed cause of p, and *pqr* the completed effect of *abcd*. The next step in our approximation to a reciprocal universal is to attempt to find the completed cause of *pqr*. Since *abcd* constitutes the completed cause of *p*, the completed cause of *pqr* will include *abcd*, but it may require further determination: *abcdef*, let us suppose, is found to be the completed cause of *pqr*. By this process we approximate more and more closely to a reciprocal universal, which may ultimately be supposed to assume the form 'Every *abcdef* is *pqrs*, and every *pqrs* is *abcdef*.' Let us retrace the steps by which we arrive at this double formula: beginning with *p* alone, we established 'Every *abcd* is *p*'; next with the conjunction *abcd* as our starting-point, we established the universal 'Every *pqr* is *abcd*'; then with the conjunction *pqr* as our starting-point we established the universal 'Every *abcdef* is *pqr*'; and lastly, starting with the conjunction *abcdef* we established the universal 'Every *pqrs* is *abcdef*.' This procedure is assumed to have reached its termination from the fact that *abcdef* implies not only *pqr*, but also *s*, so that the relation of inferability between *abcdef* on the one hand and *pqrs* on the other hand is reciprocal.

§ 7. The notion of a completed effect or a completed cause may be approached from another point of view. Taking as before *abcd* to be typical of a cause-conjunction, we shall enquire what effect-characters can be inferred wherever this cause-conjunction is manifested. Let us suppose that *p*, *q*, and *r* are three independently definable characters which can be called effects of the junction *abcd*, so that the universal proposition 'Every *abcd* is *pqr*' can be asserted. But if *p*, *q*, *r* are to be called effects proper to the cause-conjunction *abcd*, a further condition beyond the truth of the universal proposition is required. Not only must it be true that 'Every *abcd* is *pqr*,' but it must also be true that neither *p*, nor *q*, nor *r* could be inferred as effect from any cause-conjunction involving some only of the factors *a*, *b*, *c*, *d*. For example, if every *abc* were *p*, then *p* would be not an effect proper to *abcd*, because *p* would be the effect proper of the part-conjunction *abc*. Thus in order to find the effects proper to the conjunction *abcd*, we must exclude all effects which could be inferred from *a* alone, or from *ab* alone, or from *bc* alone, or from *abc* alone, etc. Or, to take another illustration of a closely connected point, the universal proposition 'Every *abcd* is *x*' would not be a true expression of causal law supposing that we could have dropped the *d*, and expressed the universal in the wider form 'Every *abc* is *x*.' Relatively to *x*, the conjunction *abcd* would contain a superfluous factor, and super-complete assignment of cause is as invalid as insufficient assignment; thus, in formulating a universal proposition stating a causal relation, the cause must not only be complete, but it must *not* be super-complete. These conditions are effected, in the case before us for instance, by taking separately each of the cause-characters *a*, *b*, *c* and *d*, and finding the effects which are due first to the factors taken one by one; secondly to the factors taken two by two; and thirdly to the factors taken three by three; and thus finally to reserve as the effects proper to the conjunction *abcd* those which can be inferred from the complete conjunction alone. Assuming then that we have standardised our causal formula by excluding those effects for which *abcd* would be a super-completed cause, we shall suppose that *p*, *q*, *r* severally are effects proper to the completed cause *abcd*. The question next arises as to whether *pqr* is the completed effect of *abc*; for just as in completing the assignment of cause we have to avoid the error of redundancy, so also—though for a different reason—we have to avoid redundancy in our assignment of the completed effect. The super-completed effect will be one in which we have failed to distinguish an effect from the effect of an effect; thus we should have wrongly assigned *pqr* as the effect of *abcd*, if *r* itself were the effect of *pq*. In other words, the completed effect must consist in an independent conjunction. Summarising the conditions, then, for the correct formulation of the causal law which presents *abcd* as the cause of *pqr*:

(*a*) The characters *a*, *b*, *c*, *d* must be independently definable and independently co-variable.

(*b*) The characters *p*, *q*, *r* must also be independently definable and co-variable.

(*c*) None of the effects *p*, *q*, *r* must be inferable from a conjunction included in, but less comprehensive than, the conjunction *abcd*; and conversely.

If *p*, *q*, *r* are the only effect-characters which satisfy these conditions, then the conjunction *pqr* may be called the completed effect of the cause-conjunction *abcd*. Any other effect-character such as *s* would have to be excluded, either because it was an effect of *pqr*, or because it was an effect of a conjunction more comprehensive or less comprehensive than *abcd*. Finally then, when the above conditions are satisfied the relation between the cause-conjunction *abcd* and the effect-conjunction *pqr* is reciprocal; so that 'Every *abcd* is *pqr*' and 'Every *pqr* is *abcd*'; moreover, both uniqueness of effect entailed by the given cause, and uniqueness of cause entailed by the given effect are secured.

CHAPTER VI

CAUSE-FACTORS

§ 1. The validity of the antithesis between nomic necessity and universality of fact being admitted, it has frequently been supposed that, within the range of the nomically necessary, causal laws can be distinguished from non-causal laws. But this view must be rejected. Causal laws have been held to apply only where *change* is involved; we have therefore to enquire into the significance of this notion, and in place of the somewhat obscure term change, I shall introduce the notion of alterable as opposed to unalterable states of a thing. This phraseology would not be admitted by those philosophers who recognise only events or occurrents, and do not allow, except for linguistic convenience, the notion of a continually existing thing to which states or occurrents are referable. I must here restate in more detail my view that any occurrent is to be referred to a continuant, and that the relation of an occurrent to its continuant, or inversely of a continuant to any of its several occurrents, is a unique relation, to which there is no analogue in any other aspect of reality. A relation sometimes hastily confounded with that of occurrent to continuant—which I will call inherence—is the relation of substantive to adjective, which I call characterisation. But since an occurrent may be variously characterised it is obvious that it stands to its characterisation as substantive to adjective; the relation, therefore, of the occurrent to its characterisations cannot be identified with the relation in which it stands to its continuant. The continuant itself might be called a substantive proper, in the narrowest possible sense of this phrase; but I include under the phrase substantive proper both the occurrent and the continuant, thereby indicating that the relation of the one to the other is not the same as that of substantive to adjective.

To define more explicitly the notion of a continuant, we will assume that any continuant has several modes of existence, or rather modes of manifestation of existence, each of which may theoretically be conceived as a determinable[1]; and according to the nature of this set of determinables, the continuant may be said to belong to one or another category. We assume further that during the period throughout which a continuant exists, every one of its modes is being manifested in some or other of its determinate forms. In the proper mathematical sense, time is of one dimension, but in order to conceive of the existence of a single continuant, it will be helpful to represent time, in a sort of figurative imagery, as having a number of parallel dimensions. Applying this figure of speech to the continuant, we may say that its existence is prolonged along a number of parallel lines of time, each of which manifests from moment to moment the several modes of manifestation in one or other determinate form.

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[1] My terminology should be compared specially with that of Descartes and Spinoza. What I call a determinable is almost equivalent to what they call an attribute, and my determinate almost equivalent to their mode of an attribute. My use of the term 'mode' will, therefore, be seen to differ from theirs.

These lines of time, therefore, are conceived as being completely filled or occupied by actual manifestations; and the conception of parallel time-lines must be extended so as to apply to all continuants.

§ 2. With these preliminary remarks I will pass to the temporal and spatial relations involved in the conception of causality. In the first place the antithesis between occurrent and continuant corresponds to the antithesis between the transient and the permanent or persistent. Popularly speaking, what exists may have only transient existence, or else persist continuously throughout a period of time, perhaps indefinitely prolonged at both ends. In attributing continued existence to a thing, we do not mean that some property of the thing continues unchanged; for a property stands to its continuant in the relation of adjective to substantive. There is a further distinction amongst properties which characterise a continuant, according as these change or persist unchanged throughout a period of time. The continuity of the existent is something behind even the possibly changing properties, and change applies not to the continuant itself, but to the adjectives which characterise it or its occurrents. Often the term cause is applied indiscriminately either to the continuant itself or to some of its properties regarded as permanent in relation to the particular occurrences or events as effects. Cause, in this sense, is essentially something persisting throughout time, and effect something essentially transient and alterable; so that the cause is not homogeneous with the effect, and this usage of the term cause is to be carefully distinguished from the notion as applied to related occurrences. Mill fails to point out this distinction when he allows himself to deal in a separate chapter with permanent causes or cause-agents, and in so doing departs entirely from his preliminary account of cause and effect as temporally related, the one as antecedent and the other subsequent in time. We must be on our guard, then, against the habit of confusing causality regarded as a relation between events with causality regarded as the relation of a permanent existent to its alterable conditions or relations. For the present I propose to confine the discussion to the more common and familiar application of this notion to occurrences. In this sense cause and effect are homogeneous; i.e. the same sort of thing that can be said about the relation of cause to effect can also be said about the relation of effect to cause. Thus if a cause process is simultaneous with an effect process, this temporal relation of simultaneity is convertible; or again if the cause process is anterior to the effect process, the latter is posterior to the former. There is a further reciprocity between cause and effect when we conceive of objective determination in its wide sense; for it is held in modern times that the specific characterisation of an effect determines the cause in the same objective sense as the specific characterisation of a cause determines the effect. This view is almost universally accepted, at any rate from the epistemic point of view; i.e. it is held that the knowledge, say, of a sufficiently omniscient being of what is customarily called the effect, would permit of inference as to the nature of the cause with just as much certainty as inference from the knowledge of the cause to the knowledge of the effect. If this be so a real problem arises as to whether ontologically, as opposed to epistemologically there is any objective antithesis between the relation of cause to effect and that of effect to cause, since each of them may be said to determine unequivocally the nature of the other.

These general considerations lead to an apparent paradox with respect to the reference of causality to time and space. Philosophers, scientists and logicians alike have often put forward as the one supreme principle of causality, that the causal dependence of event upon event is wholly unaffected by temporal and spatial differences. On the other hand the analysis of every phenomenon in terms of cause and effect assigns spatio-temporal relations between cause and effect. This paradox is removed by considering that the formula in accordance with which one event is causally connected with another, is independent of the date and location of the events, but dependent on the temporal and spatial relations between them.

§ 3. The alleged distinction between two types of objective law serves to introduce Mill's distinction between uniformities of co-existence and causal laws. The phrase 'uniformity of co-existence' requires special consideration, because it has to be distinguished on the one hand from formal universals, and on the other from causal laws. Formal universals are concerned with the spatial and spatio-temporal relations involved in the notion of movement. The difference between such formulae and those which connect the properties of continuants or the characters of occurrences, is that the latter refer to existents whereas the former do not; the term existent here being understood to apply to what is potentially or actually manifested in time, or in space, or in both time and space. Briefly the geometrical and kinematic formulae comprised under formal universals express the nature of time and space themselves; whereas uniformities of co-existence and of causation express the nature of that which occupies time and space. The latter uniformities therefore include or presuppose the former, while obviously the former do not include or presuppose the latter. Passing now to the contrast or connection between uniformities of co-existence and uniformities of causation, the two points which we shall proceed to maintain are, first, that no causal law can be formulated except by reference to co-existing properties of continuants as well as by reference to changeable occurrences; and secondly, that the required distinction is not simply one of temporal relation, such as simultaneity and sequence.

§ 4. If we consider what is involved in defining or describing an occurrence, we find that it must always entail reference to a continuant; and that one occurrence is defined as agreeing with or differing from another, by reference to the *properties* of the continuants concerned. For example, the occurrence described as drinking water is different from the occurrence defined as drinking ether, not by reference to anything which could be described in terms of actual perceptible phenomena, but by reference to the different properties or potentialities implied by the terms ether and water respectively, which denote different *kinds* of continuants. It is true that the smell and taste of ether would immediately distinguish it in sensation from water; but for a person who might accidentally have lost his susceptibility to smell and taste these perceptible differences would be unnoticed. Hence in considering the causal conditions which produce the different effects following upon the taking of ether or the taking of water, the different *properties* of these substances must be specified. Further proof of the inadequacy of the statement of causation which regards the cause as an actual occurrence related as simultaneous with or antecedent to the effect occurrence, lies in the fact that the cause assigned to account for a given effect includes not merely what has occurred in actuality, but what would have occurred under totally different circumstances. Thus the cause assigned to account for the observed effects of drinking ether would be that ether is poisonous, and this statement, though explicitly asserting the co-existence of certain properties, is implicitly a statement of causal law, presumably discovered by means of experiments in the laboratory.

§ 5. Not only is the *cause*, in my view, something more than a mere actual occurrence, but the *effect* may be something more than a mere actual occurrence; for there are many cases in which a complete account of the effect must include besides what could be characterised in an occurrence as immediate and actual, a characterisable change of property, i.e. of a potentiality that may be actualised in future manifestations. This is most obviously illustrated from the phenomena of habit and memory; for it is in mind that modifiability of property is specially prominent. Another respect in which such phenomena differ from those which are described in an ordinary account of an occurrence is that whereas an occurrence, taken as effect, is generally referred to a single completely characterised occurrence as cause, in the case of habit, the effect produced must be accounted for, not by a single previous occurrence, but by a repetition of occurrences agreeing with one another in some respect[1]. These illustrations suffice to show first, the necessity of referring an occurrence to a continuant; and secondly, the necessity of including in an account of causal conditions, properties defining the *potentialities* of occurrents, as well as characters describing the actual occurrence. My terminology may be compared with the Aristotelean classification of causes; for Aristotle's material cause corresponds closely to what I call the continuant whose nature is manifested in causal processes; and his efficient cause approximately corresponds to what I call the property, and which, when analysed as a potentiality corresponds to the Greek term δύναμις. Finally what the scholastic logicians term the 'occasional cause' is to be understood as equivalent to the occurrent cause. It should be noted, however, that such distinctions are incorrectly described as distinctions amongst causes, for they are really distinctions amongst causal factors. Thus the continuant, the property of the continuant, and the occurrence, are three factors which jointly constitute the completed account of the cause. When a causal law is expressed in condensed form as a coexistence of properties, it is absolutely essential that the term used to denote the cause should not connote a property which represents the effect, for otherwise the supposed law is nothing but a verbal proposition. For example, the proposition 'Poison always kills people' appears to express a uniformity of co-existence between the property defined as poisonous, and the property of occasioning death whenever introduced into a living person; but since this last property merely gives the meaning of the first, the proposition neither expresses a genuine law of causation nor a genuine law of co-existence.

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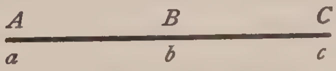
[1] This may of course be resolved into the preceding; for each single occurrence effects a change in the potentialities of future occurrences.

§ 6. Turning now to our second point of distinction and connection between uniformities of co-existence and uniformities of causation, we must direct special attention to the occurrent factors in causation, with the object of examining first the temporal and later the spatial relations between the cause-occurrence and the effect-occurrence. The typical case of causation which has figured most prominently in philosophy since the days of Hume, is where the cause-occurrence is taken to have preceded in time the effect-occurrence. The language used to express the relation of temporal sequence or succession has generally suggested the idea that the cause-occurrence can be dated at one moment of time, and the effect-occurrence at another moment of time, with a temporal interval between the two moments. Philosophical criticism has generally rejected this account on the ground that it implies, as a necessary condition for the existence of the effect, the *non*-existence of the cause. In other words, the time at which the cause, as here regarded, operates in determining the character and existence of the effect, is the time at which the cause has ceased to exist, and can therefore no longer manifest its character. This attack upon the common statement of causation has led to an attempt to overthrow the notion of causality itself, on the ground that it involves an irremovable paradox or contradiction. Now we have only to apply the above account of causation to the simplest known case of causal process to see that in truth it is fallacious. The mere datum which defines the collocation of a system of particles, would not enable us, even with the completest knowledge of the causal laws of motion, to assign their subsequent positions. The datum in this case defines an occurrence by the position at a moment of time of each of the particles; but the further datum required, in order to ascertain the positions at a subsequent moment of time, is the rate of movement of each particle within a period of time. This simple case points to the general principle for defining the temporal relation between a cause-occurrence and its effect-occurrence. Instead of dating a cause-occurrence and an effect-occurrence *at* two separated moments of time, we must define the cause-occurrence as a process going on *within* a certain period of time, and the effect-occurrence also as a process going on within a certain period of time. If the period assigned to the cause is earlier than, and not simultaneous with, that assigned to the effect, then the two periods must ultimately be taken as strictly contiguous: that is, the terminal phase of the cause-process coincides in time with the initial phase of the effect-process.



The line drawn above will serve to represent the difference between the inadequate and the adequate account of cause and effect. If an occurrence *A* be dated at the moment *a*, and an occurrence *C* at the moment *c*, and *A* be then defined as the cause of the effect *C*, the account is inadequate; for not only does it involve the above-mentioned paradox, that the non-existence of *A* is a condition for the existence of *C*, but the account fails to assign any principle for determining the interval of time which must elapse between the moment *a* at which *A* has ceased to be, and the moment *c* at which *C* is manifested. If the time-interval between *A* and *C* is phenomenally unfilled, no account can be given of its length; we must therefore represent the interval between *a* and *c* as occupied by a process of change, say from phase *A* at moment *a* to phase *B* at moment *b*, and again from this latter to phase *C* at moment *c*. We shall then no longer speak of phase *A* at moment *a* as the cause of phase *C* at moment *c*, but rather of the change from *A* to *B* within the period of time *ab* as cause of the change from *B* to *C* within the period of time *bc*; where no empty gap of time separates the cause from the effect. The cause in this case may still be said to precede the effect, but it is necessary to add that the temporal relation is one of strict contiguity. When we can quantify the differences of phase, it is possible not only to indicate the nature of the change which takes place within the whole period, but to correlate the degree of change from *A* to *B*, and *B* to *C*, with the time-relations *ab* and *bc*. In the simplest case, the quantity or degree of change is proportional to the period of time within which the change takes place: for instance, to illustrate the first law of motion, *A*, *B*, *C* might stand for successively occupied positions of a moving particle, so that *AB* and *BC* represent distances;

then the distances AB, BC, would be proportional to the periods ab, bc. In this way a principle is supplied to account for the length of time which must elapse between the occurrence of the cause and the occurrence of the effect, when these occurrences are dated at separated moments of time; and the initial paradox is removed.

CHAPTER VII

THE CONTINUANT

§ 1. We have found in our analysis of the nature and determination of occurrences that some link, besides mere temporal and spatial connection, must exist between one occurrence and another in order that the first may be conceived as determinative of the second. This link I have called the substantive continuant, and in this chapter we shall examine the notion in detail, and show how it differs from the traditional conception of substance.

The simplest and most obvious illustration of the continuant is the case of the moving particle: thus, if two movements, defined in character by direction and velocity, and defined also by reference to the period of time and region of space within which each takes place, are to be conceived as connected, in the sense that the character, date and location of the one is determinative of the character, date and location of the other, then such a connection can only be presumed if the same material continuant is existentially manifested in the two movements. Apart from the introduction of the continuant, this simple example serves to illustrate the way in which identity and difference is involved in causality. We speak of two movements, and briefly call the one cause and the other effect. Inasmuch as the movements are two, they cannot be identical; so that it may be laid down as the first and most indubitable principle of causality that, whatever other subtle relations there may be between cause and effect, the relation of non-identity is to be unequivocally asserted. Hence, before the movements in question are connected as cause and effect, they must first be distinguished as *other* or *two*; and since time and place are the only conditions of otherness which have been conceived by the human mind in regard to physical phenomena, the movements in order to be conceived as two, must occupy either different periods of time and the same region of space, or different regions of space and the same period of time, or different periods of time and different regions of space. We will suppose that the two movements, connected as cause and effect, are referred to different periods of time and to different regions of space, and proceed to consider their characterisation as regards direction and velocity. In the very simplest case afforded by science of causal relation between movements, the direction and velocity of the movement called cause is identical with the direction and velocity of the movement called effect; in this case, therefore, cause and effect are non-identical as regards temporal and spatial reference, but identical as regards characterisation. Turning now from the adjectival characterisation of the occurrences to their substantival connection, our illustration may be expressed in terms of the first law of motion, as follows:—So far as the movement of a particle within one period of time is causally determinative of its movements within another period of time, the direction and velocity of movement is the same within these two periods. Here the two movements which are causally connected are movements of *one and the same particle*; so that substantival identity is a notion essential to the understanding of the formula. It is to this substantival identity that I refer when I speak of a continuant.

§ 2. For logical purposes it replaces the term substance, familiar in metaphysics; but the various unfounded or *a priori* characteristics which philosophers have attributed to substance must be carefully separated from the essential logical residuum, and rejected from the notion of the continuant. Thus, in the first place, the conception of continuance has been extended into the infinite future and the infinite past. In my view, on the other hand, the application of continuance must be strictly limited to the periods of time in reference to which we can speak of change; that is, so far as we are justified in speaking of a state or condition as changing when we pass in thought from one period of time to another, so far are we justified in conceiving of the same entity or continuant as preserving its existence throughout the two given periods. This does not warrant us in asserting its existence either *before* or *after* these two periods. In physics, it is true that scientists have found it convenient to postulate an indefinitely prolonged existence into the past and future of the ultimate atoms which constitute matter; but this has no general logical or philosophical warrant, any more than there is philosophical or logical warrant for immortality.

§ 3. The next way in which metaphysicians have characterised any continuant or substance in an unwarrantable fashion, is by maintaining that amid all the alterations of state or condition which the substance undergoes, there are some one or more characters which continue to be manifested unchanged. This position may be held in a more or less crude form. From my point of view, what is important to point out, is merely that substantival continuance does not necessarily imply any adjectival changelessness. When philosophers like Locke and Hume sought for significance in the conception of *identity*, as substantive continuance used to be called, they were continually guilty of confusing continued existence of the same substantive entity with qualitative or adjectival identity of character or state. Failing to find this qualitative identity, Hume explicitly rejected substantival identity; and those who opposed Hume held equally with him that substantive identity could not be maintained except in so far as qualitative identity could be established. I repeat then, that the conception of substantive continuance does not by itself carry with it the implication of unchanged character through the period of time to which the substantive continuance applies. As in the matter of absolute temporal permanence, so also in this question of unchanged character, the physicists have found it convenient to postulate in various forms an unchanged continuance of character in the atoms or compound bodies which constitute the matter of the universe. Though I have here called these postulates of the physicist assumptions, I do not wish to deny that some of them may have inductive warrant; to this we shall have to return when we consider scientific induction in detail.

§ 4. The third and last *a priori* attitude towards the notion of the continuant must be briefly treated. This is the contention that the ultimate continuant is simple and not compound. On this subject it is perhaps of greatest importance at the present day to distinguish between compound, in the sense of a whole consisting of parts, and compound in the sense of involving inner causal or dynamic interaction. The former conception raises no serious problem, the continued identity of the whole being obviously involved in the continued identities of the parts. It is possible however to conceive of a compound entity which continues to preserve its identity through change of time, although none of the parts, which appear from time to time to constitute the whole, can be said to preserve their several identities. This may conceivably be explained by exhibiting a law or principle in accordance with which the compound continuant develops a changing character by means of the instrumentality of the dynamic interactions amongst the parts or components which from time to time constitute so to speak the substantival material of which the compound continuant is composed. Thus the law or principle according to which the character of the continuant at one time can be exhibited as depending upon its character at another time, may be the ground for asserting continued existential identity, although the material components of this continuant are not themselves continuant.

§ 5. We began our exposition of the continuant by an illustration from physical science, showing how the physical continuant is involved in the simple formula known as the first law of motion. We shall now bring forward an illustration of approximately equal simplicity from the psychical sphere. In the physical illustration we included reference to space as well as to time; in our psychical illustration we shall drop, for the present, any reference to space. If a sensation characterised in some way, and a thought process, also characterised in some way, occur one within some period of time, and the other within the same or a different period of time, then the character and date of the sensation can only be conceived as determinative of the character and date of the thought process if, in the simplest case, the same psychical continuant is existentially manifested both in the sensation and in the thought process. Precisely as in the case of physical phenomena, change or alteration in time does not mean the replacement of a sense-experience of red, say, referred to one period of time, by a sense-experience of blue referred to another period of time; for the mere reference of differently characterised experiences to different periods of time does not constitute what we call change or alteration. Here as in the case of the physical continuant, we can only speak of change or alteration by conceiving of an existent which continues to exist within both the periods of time to which the change refers; and it is for this reason that we call such an existent a continuant.

But the notion of change, when applied to the psychical continuant, raises a peculiar problem when we consider the different kinds of experience referable to one and the same continuant: thus we may put the question whether it is correct to speak of a change of state when, for example, we refer a sensation to one date, and a thought or volition to another date; or when we refer say a colour sensation to one date and a sound sensation to another date. The mere fact that any colour sensation is by definition different from any sound sensation, and still more that any sensation is different from any thought or from any volition, does not appear to justify us in speaking of change or alteration when such phases of experience are referred to different dates. On the other hand we should with less hesitation speak of change or alteration when the differing experiences come under the same determinable. A sensation of red followed by a sensation of blue—blue and red being determinates under the same determinable, colour—would appear to illustrate the notion of change of state more correctly than a sensation of red followed by a sensation of noise, or by a thought about geometrical relations, or again by a voluntary decision to get out of bed. Now the real reason why we apply the word change preferably to the first case and not to the second is because we suppose that the blue sensation has replaced the red sensation, so that at the time that the blue is manifested, the red has ceased to be manifested. It is thus the cessation of one character of our experience, and its replacement by another, that constitutes the essence of change.

§ 6. The above analysis helps towards a solution of the problem as to what it is that can be said to change. On the one hand, it cannot be the continuant itself, nor any of its properties, since these are asserted to be constant throughout the period of time to which the process of change is referred. Neither can it be the manifestations, dated at time-points, which can be said to change, since these merely replace one another from instant to instant. The clue to the problem is to be found in the theory of the determinable. The character of each dated manifestation is determinate, and a change implies always that the determinate character of the one manifestation at one instant is replaced at a subsequent instant by a manifestation having a different determinate character under the same determinable. Thus we speak of temperature or colour or size or shape, etc., as changing or remaining constant during a certain period of time; it is therefore the manifestation —not of a *determinate*—but of a *determinable* that may be said to change. But further, the idea of change involves not only the adjectival determinable, but also the substantival determinandum; for change would have no meaning unless there were a continuant, which was necessarily manifested in a mode characterised by one or another determinate value of a determinable. Thus the substantival determinandum is conceived as continually manifesting one or another determinate character under the same determinable, and being potentially manifestable in a mode characterised by *any* value of the determinable. This aspect of the nature of change leads to the conception of that which determines this potentiality to become an actuality; in other words the conception of change brings with it the conception of causal determination.

To prevent one minor confusion it is necessary to point out that what holds of change proper holds also of the continuance of the manifestation unchanged; for the fact of continuance as well as of change requires the assignment of a cause. The fact that the popular mind demands only an explanation of change which will assign the event or occurrence operating as cause, is accounted for by familiarity with unchanged continuance in many manifestations. Actually the preceding unchanged continuance constitutes in such familiar cases the cause of the subsequent unchanged continuance; but it is only when this continuance is interrupted that the question of the cause of interruption is generally raised. For this reason the conceptions of cause and of change are always supposed mutually to involve one another.

§ 7. The simple illustrations which we have brought forward of a physical continuant and a psychical continuant, have served to introduce the view that the two notions, familiarly known in philosophy as substance and causality, are mutually dependent the one upon the other. No adequate account of causality can be given without reference to the conception of substance, i.e. of an existent continuant, physical or psychical; and on the other hand, we can only assign properties to the substance or continuant by defining the modes according to which it is existentially manifested as a *causal* agent or re-agent. Thus what is called a property of a continuant is not an actually manifested character, but it defines what characters would be phenomenally manifested when certain assignable conditions occur. For example, the elasticity of an extensible string illustrates a property which we attribute to the string; it defines in general terms the degree of length which would be attained were the string exposed to a certain tensional force. A property, therefore, expresses a definable group of manifestations—not as actual—but as *potential.* The general formula for expressing the property of a continuant c assumes the shape: if a certain occurrence defined as p were to take place, in which the continuant c is patient, then a phenomenal manifestation defined as q would occur which is determined by the nature of c.

It should be observed that continuants—i.e. in ordinary language things—are classified according to their properties; such familiar terms as solid, liquid and gaseous, for example, do not describe phenomena as actual but as potential, and they are typical of an innumerable host of terms in familiar use. It is not altogether easy to distinguish those adjectives in common use which denote respectively actualities and potentialities of manifestation; in fact the problem of their distinction raises points of philosophical importance. As regards *physical* continuants, in predicating such adjectives as those of position, shape or size, or more generally of spatial configuration, I shall assume that we are predicating actualities; but these are the only adjectives descriptive of merely physical phenomena which are regarded unanimously by physicists as actually manifested. I myself hold that there is, besides spatial configuration and motion, another physically actualised mode, namely force as defined in statics. Physicists appear to me to maintain that, where equilibrium exists, what has been called force is merely an indication of potentiality for movement; so that only the energy of movement is *actual*, and in static condition force is held to be a myth. But in my view, static force represents a real condition of a body; e.g. when a heavy body is in equilibrium on a horizontal surface, the force called pressure actually exists, and is not a mere measure of what would take place if free motion were permitted. One evidence for this view is the recognised association of strain with stress; i.e. stress is a particular example of force which is correlated with strain; this latter being a geometrical conception. The formula according to which strain and stress are mutually connected and yet distinguished, so that they stand to one another as cause to effect or as effect to cause, appears to me to place them both in the category of actualities, since a cause cannot be said to be in operation if we conceive it as a mere potentiality. I should therefore include in what used to be called the primary qualities of matter, besides spatial configuration and motion, resistant force, this phrase being preferable, in my opinion, as well as of wider application than the dubious term 'impenetrability.' My definition of the so-called primary qualities is, therefore, that they denote the adjectives or relations in terms of which *actual* physical phenomena can be described; whereas the so-called secondary qualities are properties, inasmuch as they denote *potentialities* for producing sensational effects. Thus in describing the colour of the surface of a body, we may be defining something physically actual, but we are also most certainly defining, besides, what is merely potential; viz., that if a luminous centre, such as the sun, is in such spatial relation as to radiate energy to the surface of the body in question, then assignable parts of this energy will be absorbed at the surface, and another assignable part emitted. Correlated with this physical potentiality of the body is a psychical potentiality, which must also be presented partly in spatial terms; viz., that if a living organism susceptible to light-impressions be in appropriate spatial relations to the body, there will be a visual sensation to which the name red primarily and properly applies.

The varied applications in physics of such terms as coefficient or index are obvious illustrations of what, from the logical standpoint, we regard as potentialities in contrast to actual physical process. Such terms denote what are commonly called constants; and the common use of this term will serve to illustrate the notion of a property. Most so-called constants should more strictly be called 'variable invariables,' for only a few of them are absolutely invariable; they assume different values for different classes of bodies, although they may remain potentially invariable when applied to any one the same body. This illustrates the point that in attributing a property to a body, we imply that a certain formula can be asserted of the processes in which that body may be concerned, which formula remains unchanged on the different occasions in which the processes may take place. It further illustrates the point that bodies may be classified according to the different values of their determinable properties, as represented by the different values of the so-called constants. As an example of this, let us take the adjective 'ruminant.' So far as we predicate this adjective, we are certainly implying the existence of an organ of specific character, which could be defined anatomically or structurally in terms of spatial arrangement and, say, mechanical pressure. In addition to these elements of definition, we should further assume that the organ is persistently functioning, though the determinate mode in which it is functioning would be changed from time to time in accordance with a general formula defining the potentialities of the organ to function whenever assignable conditions may be actualised.

§ 8. So far we have been discussing the continuant chiefly from the point of view of deduction; but I propose now to treat it from the point of view of primitive induction, and to exhibit a constructive process by which, prior to anything that could be called classification of organised or of unorganised bodies, mankind have put phenomena into groups. The symbols P_1, P_2, P_3... will represent phenomena characterised by P, the suffixes being understood to indicate some kind of *order*—either temporal or spatial or both—which we shall speak of as a nexus. The first motive for grouping phenomena is the observation of some such nexus: thus when an order from P_1 to P_2 to P_3 to P_4... has been repeatedly observed to be maintained under a variety of circumstances which, in some of the different recurrences have been constant and in others have varied, then these phenomena have been grouped as manifestations of an existent agent. This observed uniformity in the temporal succession of phenomena is then inferentially extended to apply to other assumed phenomena, regarded as modes in which a single agent manifests its continued existence. The above conception of nexus therefore involves not only a preserved temporal order of phenomena, but also reference of these phenomena to a single continuant. In physical, as distinguished from psychical, phenomena there is, in addition to the temporal nexus, a spatial nexus, between phenomena presented in spatial contiguity, representing modes in which a single material body or occupant is spatially manifested. Thus when the spatial order of such characters as P_1, P_2, P_3... is preserved throughout exterior processes which are changing or remaining unchanged, the group of characters is with special impressiveness taken to constitute a unity, and conceived as referable to a single occupant which maintains its form of spatial nexus, however exterior conditions may be altering.

The unity of the occupant is, however, not stably maintained with the same degree of permanence as is attached to the temporal succession amongst the manifestations of the continuant. In this respect spatial occupants fall into different classes according to the degree in which they preserve the form of their spatial nexus: the bodies which preserve their spatial nexus in the highest degree are called solids; those that preserve it in less degree are called liquids; and those in which the spatial nexus can hardly be said to be preserved at all are called gases. These degrees of spatial nexus actually depend upon the power of bodies to sustain tension and pressure; bodies which can sustain both are called solids; liquids cannot sustain tension, but only pressure; and gases, which are also unable to sustain tension, would lose their spatial nexus altogether unless pressure, produced by external force, determined the space which they can be made to occupy. Bodies regarded as occupying spaces of defined shape and size can be divided into classes on many different *fundamenta divisionis*: for instance according as they are unorganised or organised in various different forms, or again according to their chemical composition. A body is classified according to the mode in which it maintains its unity as a whole, this unity consisting in the *permanence of mode* of presentation of its manifestations, while exterior conditions change or remain constant. When the spatial unity which leads us to conceive of a body as a single whole is dissolved by exterior force which the body is unable to resist, then each of the several parts into which the body becomes separated has its own constitution as a unit; and this constitution may or may not be generically the same as that of the whole into which the parts were previously united.

§ 9. The consideration of the different degrees of spatial nexus exhibited by different spatial occupants, leads to the notion of a single continuant-occupant being at the same time a system of *sub*-continuants or atoms. These sub-continuants constitute a system, in the sense that they preserve a certain form of spatial nexus either unchanged, or else changing in accordance with a causal formula which expresses both the immanent causality to be attributed to the several sub-continuants, and the transeunt causality to be attributed to the interactions amongst these sub-continuants. For example, the gas that is contained in a vessel is a system of sub-continuants—viz. the gaseous molecules—which severally exhibit their own immanent causality, while their spatial nexus, manifested as mutual pressure, exhibits transeunt causality. Again each molecule may be a sub-system consisting of sub-continuants—viz. atoms—which taken severally exhibit immanent causality, and taken in combination, transeunt causality. Thus, any system of sub-continuants may be regarded either as a unity or whole, whose changes are determined under the form of immanent causality; or any such whole may be conceived as a system of parts which are themselves continuants, and whose processes are to be distinguished from the transeunt processes involved in their interactions in the larger system. In future, the phrase 'single continuant' is to be understood to include a single system of sub-continuants or sub-occupants. Such a system may be ranged in an order exhibiting higher and higher forms of unity; of which the lowest form is a mechanical system, and the highest a psychical continuant. In a mere mechanism, the system as a whole can be defined in terms of the immanent character of its parts, combined with the transeunt causality under which the parts interact; but the higher form of unity exhibited in an organism entails a much more complex interrelationship of immanent and transeunt interactions; thus each part of an organism undergoes processes which follow chemical and mechanical formulae, which remain true even when the organism is dead.

§ 10. The forms of temporal and spatial nexus observed prior to experiment to have been maintained between groups of phenomena, lead to their constructive reference to a single continuant-occupant. Primitive induction consists in the more or less tacit inference that where such forms of nexus have been found to obtain within the range of observation, the same form or degree of nexus will be maintained in all cases that have not yet been observed. This may be shortly expressed in the proposition: 'All manifestations of a given single continuant will assume a specific form of nexus'; and the assignment of a substantive-name, therefore, to any manifestation in space and time, is tantamount to a statement of uniformity. It should be specially noted that the range of uniformity here asserted does not extend beyond the manifestations of a single individual continuant. Now this uniformity is not a mere statement of the invariability by which we can infer from certain phenomena other phenomena in spatial and temporal connection with these, but it involves also a statement of causal relation between the phenomena. The causality affirmed is immanent, and is conceived as exhibiting the nature of the individual continuant itself. The phenomena in question may be said to *co-exist*, the prefix *co* indicating that they are manifestations of a single continuant; for, as I have elsewhere pointed out, the phrase *co-existence* of properties does not mean merely temporal simultaneity in the manifestations, but includes precedence and subsequence, and also in some cases spatial relations.

§ 11. The assertion of causality does not carry with it any implication of a *uniformity* of co-existence, affirming that the causal formula which applies to the continuant under consideration would hold of other continuants. The nature of any given individual is exhibited in a uniformity of its manifestations, which again, in my view, is a causal uniformity. But this species of uniformity is not one that can be extended in application to the manifestations of other individuals, except so far as we have inductive evidence that the same form of immanent causality that expresses the nature of one individual expresses also the nature of other individuals. The *causal uniformity* which obtains between the manifestations of a single individual continuant is established by more or less direct and simple problematic methods; whereas in order to establish a *uniformity of co-existence*, the criteria of probability employed are much more exact and logically elaborate. What I have now to maintain is that there are many conclusions reached by the first method which can be said to have been established with a very high degree of probability, amounting in some cases to practical certitude. They assert persistence in the form under which the manifestations of a *single* continuant may be subsumed. Induction, on the other hand, which attributes the same form of property to one and to another continuant, cannot attain a high degree of certitude by simple problematic methods; for what enables us to construct a class of continuants so defined, is the employment of more or less *scientific* method.

§ 12. We will now pass to the consideration of the relation of the continuant or supreme substantive to its occurrents or modes of manifestation. These modes of manifestation have variously defined characters and are connected with one another by temporal and spatial relations. Just as the supreme substantive is called a continuant on the ground that it continues to be manifested throughout an indefinitely prolonged period of time, within which any one manifestation or occurrent is referred to a specific sub-period; so it may also be called an occupant, on the ground that its manifestations occupy an indefinitely extended region of space, within which any one manifestation is referred to a specific sub-region. The determinate sub-period and sub-region together define the relation of one occurrent to others; whereas the time and space, of which these sub-periods and sub-regions are respectively parts, may be predicated referentially to the continuant-occupant itself. Thus when we say that a continuant exists throughout the whole of time, or an occupant throughout the whole of space, we mean that its several different manifestations are to be referred to one or another of the sub-periods or sub-regions within this whole; and if we are concerned with an animate or psychical continuant, the whole of time or the whole of space within which it exists may be itself a sub-period or a sub-region within a larger whole; since there may be some part of time during which it is not manifested in any part of space, and there may be some part of space in which it is not manifested in any part of time. This is one of the ways in which predications (viz. temporal and spatial), which are primarily attached to existent *manifestations* are transferred to an existent *continuant* or *occupant*. We are thus led to enquire what other adjectives can be predicated of a continuant which are derived from the characterisations of its several occurrents or manifestations. To explain this we must conceive of the unity of the continuant as exhibited in causal formulae symbolisable as follows:

$$P = f_P{}'X, \quad Q = f_Q{}'X, \quad R = f_R{}'X, \quad \ldots\ldots$$

where each equation represents a property of the continuant. The small letters f indicate that the property or function is defined determinately, while the capital letters X, P, Q, R, indicate that this same determinate property is exhibited for all determinate values which X, P, Q, R may assume at any specific time or place. Now as regards physical continuants, there are many properties or functions which exhibit literally the same determinate value at any time or in any place; but, in more complex organic continuants, such properties as have been denoted by the symbol f change from time to time. These temporal changes in the manifested properties of things are not irregular, but follow a law (dependent upon the immanent character of the things) which constitutes a property of what may be called a higher order than the properties symbolised by f, which directly define the causal characters of actual manifestations. In the case of complex organic continuants, therefore, any actually manifested property symbolised by a determinate f comes within a determinable F which, in its variations, exhibits a determinate property, say ψ, of a higher order. The formula for this higher order of property will depend upon the variable—time—as also upon the different circumstances which have operated transeuntly upon the continuant from time to time. The formula itself will, however, be a formula of immanent causality, exhibiting the nature of the continuant itself as determining the kind of effects which will be produced in it under the influence of external circumstances. It is impossible adequately to represent this notion of alterable properties in symbols, but I suggest the following scheme for illustrating the conception:

$$P = \psi_P (X,\ Y,\ Z, \ldots \quad T_X T_Y T_Z \ldots),$$

which degenerates into $P = f_P (X,\ Y,\ Z, \ldots)$ when the T's have some determinate values. From this formula the value of P is determined from the circumstances X, Y, Z, and the times T_X, T_Y, T_Z, at which these circumstances have operated. In this formula the capitals represent variables: of these variables, those in the bracket are *independent*, while the variable P is *dependent* upon these independent variables, of which it is a function. The small letter ψ represents a property of the higher order, having a determinate value which is constantly manifested whatever variations the variables may assume either actually or hypothetically. By the applicative principle, we infer from such a formula the actual mode of functioning of the continuant, when the determinables are replaced by given determinate values. The specific form of the function ψ, as indicating the character of the continuant itself, illustrates immanent causality; but so far as the circumstances X, Y, Z are due to the operation of external agents, transeunt causality is involved in occasioning the actual values that they may assume from time to time.

§ 13. From the above attempted explanation of the notion of property, it is but a short step to the concept of a continuant; for the main element in the notion of thing or continuant is the permanency of functioning that can be discerned in a series of characterised manifestations, presented in the course of time, as they may be observed in a temporally continuous, or discrete, series of acts. Thus the notion of a continuant is constructed in terms of temporal connection and causal determination, and my particular views on this subject may perhaps be best explained by comparing my account with Kant's exposition in the *Critique of Pure Reason*. Kant holds that there are certain categories, such as substance and causality, under which we objectify our sense-experiences in an order of time; whereas I prefer to treat substance and causality, not as two separate categories, but as two aspects of a single principle of construction. Again, instead of adding a third category—namely reciprocity—to substance and causality, as Kant does, I include reciprocity in my account of immanent and transeunt causality. But such apparent differences between Kant's exposition and mine are not important, and readers of Kant, by putting together various parts of his exposition, would find at least hints of all that I have said. Thus, when he says with regard to his category of substance, that the idea of change involves the idea of permanence, and when this is supplemented by his schematism of causality under the form of time, his view is seen to be in close accord with my account of the way in which temporal causality and permanency of functioning ente in the notion of a continuant; although the postulate of permanency refers in Kant's exposition to a 'quantum' rather than to the *mode of functioning* which I attribute to the continuant. In my view this permanency in the mode of functioning is inseparable from the property—or form of causality—this form being just that to which permanence is attributed; whereas Kant appears to affirm that the substance itself exists permanently, and that a second permanence is to be attributed to one of its modes of manifestation, namely to its 'quantum.' This postulate of his was, in fact, an anticipation of the constancy of mass which is a special postulate in physics; but no similar quantitative constancy can be attributed to the higher substantive entities, such as the organism or the experient. I am inclined to attribute Kant's denial of the possibility of rationalising psychology to his rather exclusive consideration of the forms in which the principles of physics can be generalised and formulated in precise mathematical conceptions. Thus my account presents a more general conception of substantive continuance, which applies equally to the notion of a conscious experient on the one hand, and to a hypothetical physical atom on the other. The unity which I ascribe to the continuant is a causal unity of connection between its temporally or spatially separated manifestations; an observed or assumed causal formula, under which the character of these manifestations may be subsumed, is the sole ground for regarding them as manifestations of one and the same continuant. I have also attempted to render clear the difficult conception of the union of permanence with change. It is natural

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to ascribe change to the modes of manifestation, and permanence to the substance to which these manifestations are referred; but this is an inadequate expression of the antithesis; for, to express the matter accurately, the only things which can be said temporally to exist are the manifestations themselves; thus our first definition of the continuant is that it is merely the sum of all the manifestations. This of course does not mean that manifestations of reality are taken indiscriminately, mentally added together, and their sum called a continuant; what is meant is that certain manifestations of reality, between which a unique kind of relation can be predicated, together constitute a genuine whole or unity, to which the name continuant may be given. This type of relation, which constitutes the unity of a single continuant, is conceived primarily as one of immanent causality, while it is transeunt causality that constitutes the ground for asserting a plurality of non-identical continuants whose manifestations can be said to belong to one universe of reality.

§ 14. All the conceptions expounded in this chapter are virtually denied by a school of philosophers to-day. In particular they regard the conception of change as fictitious, and substitute for it merely differently characterised phenomena referred to non-identical dates. Whenever there is a spatio-temporal nexus between phenomena, the locating and dating of the occurrents is such that these may be conceived as a whole. Such a whole is of the kind which we have described as extensional[1], and so far as extensional wholes are admitted by the scientist, no more transcendental conception than that of a whole constituted by the binding relations of time and space is required; and hence the philosophers who reject the conception of a continuant are satisfied to replace it by the notion of such an extensional whole. But the stability of a spatio-temporal nexus cannot, I maintain, be explained without the conception of a continuant, which, in my view, is *a priori* in the Kantian sense, and not derived from the analysis of experimental data. Given the conception, however, it is a question of mere experience to what set of phenomena the *a priori* notion is to be applied. In attempting to avoid this conception, it appears to me that my opponents alternate between a purely physical and a supposititious perceptual account of the facts. Thus in one breath they shelve the physical continuant by supposing that the percipient is observing a continuity in the qualitative changes of the object perceived; and while in this way rejecting any physical continuant, they have recourse to a psychical continuant—namely the percipient. Here I submit that the perception by any individual of certain processes offers no explanation whatever of what in objective reality determines the stability of any given nexus. Then again, on the other hand, when it is urged that the upholders of this view are all along assuming a psychical continuant—viz. the percipient—which from their standpoint must be repudiated, they, in effect, retort that it is quite unnecessary to postulate any psychical continuant, inasmuch as the nervous system itself will take the place of the ordinary conception of an ego. Here then they only eliminate the psychical continuant by reinstating the physical continuant.

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[1] Part II, Chapter VII, § 8.

CHAPTER VIII

APPLICATION OF CAUSAL NOTIONS TO MIND

§ 1. THE science of psychology—so far as it is purified from all reference to the physical—uses the conception of immanent causality within the systematised unity constituted by a single individual experient. In other words, pure psychology abstracts from any transeunt causality which may be actually operative in the interactions between an experient and the material world, or between one and another experient. The different phases within the experience of an individual are conceived as related temporally and not spatially; hence the form of space under which we conceive transeunt causality in the physical universe, does not apply within the individual's experience. A modified form of transeunt causality is applicable however to the interactions amongst the distinguishable phases revealed by a fundamental analysis of conscious experience. If, for instance, we distinguished between merely sensational processes, on the one hand, and active or purposive processes on the other, we might establish some kind of uniformity which would determine the course of a sensational process so far as it was uninfluenced by active purpose. Again a process of deliberation might be known to pursue a course of its own independently of the changes occurring in the sensational process. In this way a relative and partial independence might be attributed to the sensational and deliberative processes respectively, each proceeding according to its own law. But, if we suppose that at certain periods the deliberative process exerts a determinative influence upon the further course of the sensations, this influence is analogous to transeunt causality, with the difference that the causality is not conceived as passing across from one to another substantive existent, but only from one to another phase of experience within the unity of a single substantive existent. Another rough illustration of a similar type may be taken from associative as distinguished from attentive processes. Thus the forms assumed in the flow of images or ideas may be supposed to depend essentially in the first instance upon the time order of past experiences, and the frequency or recency of these experiences. In so far as this is so, the course of thought has a law of its own which operates independently of purposive control. But at such periods, when the felt interest of the thinking subject modifies the course of associations, and determines the flow of images or of ideas to be other than they would have been as the result of mere association, a form of mental causality is operative which is more or less analogous to transeunt causality. Those psychologists who explicitly attribute activity to the subject may be said to use the conception of transeunt causality in an even more literal sense than that which I have so far suggested : for they hold that, apart from subjective activity, mental processes would pursue a course determined on principles quasi-mechanical, these quasi-mechanical processes, constituting a sort of non-ego. Over against these, the subject or true ego is conceived as an agent having the power, which it exercises from time to time, of controlling or modifying the processes which apart from such activity would proceed purely mechanically. This splitting up of experience into mechanism and active control tends, in my view, to misrepresent the case, if it leads to a conception of the subject as purely abstract. The subject as active must be conceived as a determinate phase of experience, which stands from time to time in definable and alterable relations to the processes that may be said to be actively controlled. The two most fundamental of these relations are called respectively feeling and cognition; according to the mode in which experiences arouse feeling, and according to the manner or extent in which they are cognised, so is the exercise of controlling activity determined. Thus feeling and cognition operate as psychical forces which are analogous to physical forces, except that the latter involve spatial relations.

§ 2. Another quite unambiguous example of transeunt causality is the action of the psychical on the physical, and the apparently simultaneous action of the physical on the psychical. We may venture to speculate that various phases of psychical process which proceed contemporaneously with physiological (and in particular neural) processes, can be described in terms of the same number of distinct determinables as the neural processes. It is therefore theoretically possible to predicate of such phases of mentality a one-one correspondence with the neural processes, where the term "one-one correspondence" is understood as equivalent to reciprocal inferability. This means that when (if ever) psycho-physical knowledge has been adequately advanced, it will be possible from a knowledge of the character of any neural process that may have occurred, to infer the character of certain of the contemporaneously occurring phases of mentality, and conversely. This will give a restricted validity to the conception of *neutral* psycho-physiological parallelism, the word "neutral" indicating that the theory does not prejudge the question whether the assertion of correspondence implies an assertion of causal connection or not. The conception is *restricted* in the sense that, besides the phases of mentality which correspond in their changes to the changes of neural process, there are other psychical phases to which no changes of neural process correspond. If the former be provisionally called sensational experiences, I hold that these are to be clearly distinguished from cognition and feeling, which constitute the fundamental aspects of psychical process to which no neural processes correspond, and which may be provisionally defined as variable or alterable relations or attitudes towards the sense-experiences. This may be otherwise expressed in the assertion that there is no *direct* correspondence between the phases of cognition and feeling on the one side, and changes of neural process on the other side. For, if we assume that the changes of sensation are caused by changes of neural process, it follows that, in so far as these changes of sensation determine changes in the phases of feeling and cognition, there will be *indirectly* a correspondence between the latter and the neural processes. The more rigidly we insist that the phases of cognition and of feeling are occasioned in their changes by the changes in the sense-experiences to which they attach, the more clearly shall we realise the fundamental distinction between the *direct* correspondence of the sensations to the neural processes, and the *indirect* correspondence of these latter to the phases of cognition and feeling. If, for instance, the changes of neural process are regarded as the sufficient causal determinants of the changes in the sense-experience, and these changes of sensation as sufficiently accounting for the cognition and feeling attaching to them, then there can be no remaining modes corresponding to the phases of active cognising and feeling in which the neural processes could be described as changeable. It appears to me that all the experimental work, which endeavours to establish laws connecting feeling, for example, with sensational or physiological changes, adopts precisely this same hypothesis. In these experiments an attempt is made to formulate in general terms the sort of character which the physiological processes must have, in order to account for the accompanying feeling as being more or less pleasurable or painful. It is assumed in these experiments that all the changeable modes of *neural processes* have as their correspondents changeable modes of *sensational experiences*. There is never any hint that the physiological processes could be changed also in some further mode corresponding to the changeable phases of feeling. Similarly in the case of cognition, where by this term is meant—not merely awareness of a sensation but—the cognising it as having a certain character. Thus, when I speak of changes in the cognitive phase, I mean to refer to such changes as apprehending one sensation as red, and apprehending another as blue. In this case again, physiologists implicitly assume that all the modes in which the neural processes can be described as changeable are exhausted in accounting for the sensations *being* red or blue. There is therefore no residual mode according to which these neural processes could be supposed to vary or change which might correspond to the psychical fact of *apprehending* the sensation as being red or as being blue. A closer examination of the nature of cognition and feeling will further confirm this view; for, while the sensational processes may be supposed to be entirely accounted for by the actual neural processes that are contemporaneously occurring, the cognition and feeling which attach to these sensational processes are partly determined by past and possibly remote experiences, and therefore cannot be wholly accounted for by the present sensational and contemporaneous neural processes.

§ 3. A serious objection might be taken to the above account, on the ground that I have omitted any reference to those more central cerebral processes which physiologists describe as underlying the active phases of cognition. In this connection I will introduce the word effort or strain; and I wish to suggest that the phenomenon of effort constitutes a link between the physio-sensational mechanism on the one hand, and the subjective control which is exerted upon this mechanism on the other hand. I maintain that the phenomenon of effort or strain has both a physiological and a sensational aspect: that is to say, it involves a process in the physiological mechanism which, like other physiological processes, entails a corresponding sensation; it is therefore legitimate to speak of an effort-sensation, and the term indicates that there are certain analogies between this kind of modification of conscious experience and other sensations such as visual and auditory. Thus, there may be differences of intensity and even of quality in the experiences of effort, correlated with differences of intensity and of locality in the underlying processes of the neural mechanism. Again, an effort-sensation resembles other sensations in the further respect that at any moment it may be more or less determinately characterised in an attitude of cognition; and this act of characterisation may reach such a limit of indeterminateness that we should say in ordinary language that the sensation was not being cognised at all. Lastly, a character may be attributed to these sensations apart from any cognition of it by the experient; that is to say, this character is unchanged by his more or less determinate cognition of it. In all these respects, effort-sensations resemble what are ordinarily known as sensations; there are two important points, however, in which they differ. Briefly these are (1) that they are subjectively initiated; and (2) that they entail directly changes in the neural processes which indirectly produce effects *intended* by the subject—the term 'intention' implying foreknowledge. It is important to point out that the subject does not of course know what sort of *neural processes* are taking place, although to him is to be attributed the initiation of these processes. The sense in which his activity is guided by knowledge is expressed by his foreknowledge of the effects upon his future sensations or perceptions which will be causally determined by the particular operation that he initiates in the neural processes. Thus we attribute to subjective initiation various physical effects which, for our present purpose, may be roughly divided into two classes, describable as inner and outer respectively. The inner physical effects are those which are *unknown*, the outer are those which are *foreknown*. As physical occurrences, the inner are causally determinative of the outer; but, in our analysis of the mental processes involved, we have to maintain, what may appear paradoxical, that it is the *foreknowledge* of the outer which causally determines the *occurrence* of the inner. One, and perhaps the most fundamental aspect of this foreknowledge is when the outer effects are imaginatively characterised by the subject in their *spatial* relations. The terms inner and outer may be taken almost literally as defining what has been going on *inside* the organism, as distinguished from what will be going on *outside* the organism. When the intention of the subject has been actualised, what he perceives is presented as (so to speak) external in its spatial relations, and it is these externally manifested effects which are foreknown or prospectively imaged.

§ 4. We have said that the operation upon the neural mechanism, which appears to involve strain or effort, is initiated by the subject. In using this phrase I am not thinking of the subject in a merely abstract sense, or as a sort of transcendental ego (to use Kant's phrase); but, on the contrary, of a process precisely definable in terms of mental phase. In fact, as I have already attempted to show, it is a mode of feeling and a form of cognition which jointly determine the specific operation upon the mechanism. Under the various modes of feeling I wish to include not only the hedonic variations of pleasure and pain, according to which experiences are felt as more or less pleasant or unpleasant, but also the modes of conation, according to which we feel more or less strongly attracted or repelled by different experiences. Again cognition should be understood to include not only the foreknowledge of the finally intended effects, but also (as higher forms of knowledge develop) the foreknowledge of the external means which must be employed to produce these final effects; the special term purposive is used to describe voluntary action of this higher kind. From the above analysis it is clear then, that in the psychical determination of physical effects, foreknowledge is involved, and we have attributed to foreknowledge real causal efficiency. If this foreknowledge could be reduced to merely physical or physiological terms, we should have to regard mental causality as an illusion; and those psychologists who hold that the changing phases of cognition are represented by corresponding neural processes, do in effect deny any genuine validity to the conception of mental causality.

§ 5. Closer examination of subjective activity introduces the notion of attention; for I hold that the most important consideration in any account of cognition is the different degrees of determinateness with which the character of an object may be apprehended in an attitude of attention. It is implicitly maintained by some psychologists that a sensational process can only properly be said to occur when the subject is cognising the sensation; so that where there is *no* such cognition, these psychologists maintain that there is no sensation; and all that could be asserted (as corresponding to the non-occurrent sensation) would be a neural process which could be defined by the physiologist. Now, whether this contention is sound or not, it must be agreed by all that, when the character of a sense-experience is the object of cognition, it may be cognised by the subject with very different degrees of determinateness or indeterminateness. It will also be almost unanimously agreed that the function of attention is to render the object attended to more determinately cognised; and that continued attention to one and the same object does, in effect, produce this result. Increasing determinateness of cognition might thus be marked off as the *effect* of attention. When further we say that attention involves activity, and attribute this activity to the subject itself, we are attributing the *cause* of the process to the agency of the subject. But this does not explain why a more determinate knowledge of one object rather than of another is being developed; and to account for this a defined purpose is to be attributed to the subject, the achievement of which demands this further determinate cognition as means. It would be artificially formal, however, to draw an absolute line of distinction between means and end; our attention may, for instance, be momentarily diverted to an intrusive sense-experience, in which case the more determinate characterisation of the intruder may be an end in itself of the momentarily diverted attention, and not, at the time, pursued in the achievement of any more remote end. Incidentally it may be observed that if the sense-experience still continues, when attention has ceased to be diverted to it, it may operate as a disturbing factor in the attention which reverts to its previous object.

We are now in a position to distinguish two types of subjective activity. (1) The operation of the subject upon the *sensational* processes, and directly or indirectly upon the external physical effects presented to perception, discussed in our previous general account, may conveniently be called motor, because its actual manifestation in the physical world involves a change or maintenance of spatial position. This motor activity which produces or prevents overt physical movement is to be distinguished from (2) the activity of attention, which appears to involve only a furthering or development in the determinateness of cognition. Either of these forms of activity may entail a more or less intense effort: corresponding to the first we may speak of motor effort, and corresponding to the second attentive effort; and both motor effort and attentive effort may have a quasi-sensational as well as a neural or physiological side. In actual and overt movement we can trace the neural processes which entail corresponding sensations; but in the case of mere attention, where there is no overt movement, we have to consider what may be the nature of the physiological processes which underlie the effort-experience that seems often to accompany attentive processes. In motor activity we assumed an operation upon the neural processes underlying *sensation*; in a similar way, we may perhaps assume that the activity of inner attention entails an operation upon the neural processes underlying *imagery*. If this hypothesis is to be consistently applied, our account of "restricted parallelism" must be extended to include a strict correspondence—not only between sensational experiences and their neural accompaniments—but also between image experiences and *their* neural accompaniments. This may be done without prejudging the physiological question, irrelevant to our present purpose, whether, underlying any sense-impression and that sense-image which is a sort of copy of it is the same neural process in a different form, or locally distinct neural processes. The suggestion I wish to put forward is that such effort as appears to be experienced in inner thinking is due to the occurrence of imagery entailed in operating upon the neural processes. Without discussing this problem, I may point out a possible confusion between what is properly to be called the *effort* involved in thinking, and the difficulty of such thinking. For a given subject, the difficulty of making further progress in thinking on a certain topic—so far from implying intensification of effort—may lead him to cease thinking further on the subject. This leaves the problem still to be examined whether, so long as there *is* a continuance of the thinking process, more or less effort is involved. Evidence of such effort may not easily be found in direct introspection, and may have to be sought indirectly in such effects as a diminution in physiological or intellectual energy; i.e. in future power of doing work.

§ 6. I will now give, more explicitly, my grounds for the hypothesis that the changes of cognitive phase have not counterparts in the changes of neural process; considering in succession the three main features characteristic of even the simplest cognitive process.

(1) We may agree that perception denotes essentially the mode of cognition in which objects are apprehended in their temporal or spatial connections or relations. We may further agree that corresponding to the relations of time which we predicate of objects cognised, there are actual relations of time subsisting between the

neural occurrences which underlie the experiences whose temporal relations are apprehended in perception. The dating and temporal measuring of these neural processes by the physiologist would exhaust what could be stated about them, when their mode or locality had been defined. There appears therefore to be no discoverable further mode of neural process which should correspond to the mental act of cognitively defining the temporal relation. What holds of time, holds of space: the spatial relations between the several areas, of which the stimulation causes sensations, may be assumed to vary with the spatial relations as apprehended in the object perceived, in accordance with some formula. And the physiologist may also correlate the locality of the stimulations of the end-organs with the modes of neurosis or the cerebral localities of the physiologically central processes. This again would exhaust his account of the physiological processes, and there would be no other mode of variation which could be correlated with the act of cognitive spatialisation.

(2) Consider next the feature of elementary cognition which is involved in the act of comparison. In the simplest case this act is a cognitive determination of a relation of difference or of agreement. We may assume that the respects in which our sense experiences agree or differ correspond to the respects in which the underlying neural processes agree or differ. But as in the case of spatial and temporal connecting, there does not appear to be any further mode in which the neural processes could vary which should correspond to the act of apprehending such relations of agreement or difference; these might or might not take place without

affecting the agreements or differences themselves. This same argument applies even if those theorists are right who maintain that the only way in which we cognise the *characters* of our experience is by cognising relations of agreement or of difference between one experience and another; for here, as in the previous case, there can be no physiological correlate for cognition in general. If we pass for a moment to the higher forms of cognition which constitute the special province of logic, such relations, for instance, as those of identity, of substantive to adjective, or of cause to effect, it is still more impossible to conceive of modes of variation of neural process which could be correlated with the occurrence of such acts of conceptual cognition.

(3) The third feature of the cognitive process to which I wish to draw special attention is the variability of the degrees of determinateness or indeterminateness with which an experience may be cognitively characterised. This feature of cognition may be said to include the features treated under the two previous heads, and to constitute the most fundamental ground for our denial of a physiological correlate of cognition in general. For the cognitive determination of temporal relations, or relations of agreement and difference, etc., is to be regarded as a case of the further determination of relations which were earlier apprehended in a comparatively indeterminate form. Now the actual experiences, and consequently the corresponding neural processes, are *in fact* determinate in character; so also are the relations amongst them. How, then, can this determinateness of character and of relation be combined with varying degrees of indeterminateness that should be correlated

with the varying degrees of indeterminateness of cognition?

§ 7. It appears to me that the reason why physiologists and psychologists never properly face the problem of the neural correlate of cognition, is because they virtually identify ideas with images. This confusion is especially apparent in discussions about so-called 'general ideas,' 'abstract ideas,' 'conception,' or 'conceptual thinking.' The older nominalists denied the possibility of such forms of thought or ideation, and maintained that the only mental content which can actually be asserted in abstract thinking is the *word* heard, or uttered, or represented in imagery. For modern psychology the problem of the relation between language and thought is still a burning question. So far as language is concerned, the simplest case to take for illustration would be that in which we characterise a sense-experience, say as being red, and, in this act of characterisation, utter or image the *word* red. This process is partly explained by the formula of association, and the associative process may be safely assumed to have a neural correlate; so that the mental association between the sensation red and the word red may be correlated with some connective process, taking place between the central or cerebral processes which underlie the visual sensation and the word-imagery respectively. But even in this simplest case, it must be pointed out, that to define the mental process as merely an association between the word and the sensation, is wholly inadequate. On the occurrence of the sensation, it is not only the image of the word which is aroused in the mind of the thinker; but he mentally connects the word with the sensation in a form which could be expressed in some such proposition as: 'The word red stands for the quality characterising this sensation.' In fact, passing from this simplest case to the higher forms of thought which may be accompanied by language, the verbal expression of a proposition may be taken to represent the universal *form* of an act of thinking. Association and its probable neural correlate are involved in so far as the words, comprising the entire sentence, can be said to be associated with one another and with the objects to which they refer, so as to constitute a whole. But, if we examine the mental process, we find that the sentence is not merely a whole for the thinker, but a *significant* whole. Mere association might give an adequate account of a combination of words which was mere nonsense; it cannot account for the added psychical fact that the sentence is understood as having meaning. Now an understanding of the meaning of language, and in particular of the sentence as denoting a proposition, is what is meant by thought or ideation. If the physio-psychical process of association is adequate to explain how the words of the sentence come successively before the mind, what can be the physiological process correlated with the act of understanding its significance?

§ 8. The distinction between physio-psychical processes and those which I attribute to subjective initiation is best illustrated by those reflex processes which entail various forms of consciousness, for here the contrast is sharp and unmistakable. Let us take for example the condition which we speak of as a tendency to sneeze; this is a physiological condition which is certainly accompanied by several definable phases of consciousness:

(1) There is in the first place a characteristic sensation which is distinguishable for instance from that accompanying the physiological tendency to cough.

(2) Like other sensations, at the time of its experience its character may be cognitively defined both as regards quality and locality by the experient; and it will be so defined if any interest or purpose prompts him to direct his attention to it.

(3) So long as the physiological tendency to sneeze remains a tendency, there is an element of feeling which could be called discomfort.

(4) There is normally anticipatory imagery or precognition of what will almost immediately occur sensationally when the sneeze actually takes place. This foreknowledge of the prospective sensation involves auditory and other forms of imagery.

We may now pass from what is a direct analysis of the modes of consciousness accompanying the physiological tendency, to a consideration of the causal processes which may follow. In the absence of any inhibitory act on the part of the experient, the sneeze will take place; and in this case the causality involved is purely physical or physiological. But the sneeze has a *sensational* as well as a physiological aspect; and to this aspect the term *conative* has been mistakenly applied; the term conation being taken as equivalent to felt tendency. But this definition appears to me to involve confusion; for the phrase 'felt tendency' has been used to describe the form of conscious experience which accompanies a process which will normally terminate in an explosion; whereas it is generally agreed that *conation* is a form of consciousness of which the causality is psychical and not merely physiological. In the case of the sneeze, the word 'tendency' alone would express a merely physiological fact; and when conjoined with the word felt, it can signify only a precognition of the subsequent mode into which the sensational process accompanying the physiological fact will develop. In the physiological case, discomfort and foreknowledge play no part as *causal* factors in determining the further process; psychical causality would only enter therefore in an effort to inhibit, repress, retard, or moderate the sneeze. The exercise of this effort is conditioned by precisely the same factors as operate in any case of voluntary effort; namely by desire for a more or less specifically cognised end, and a knowledge of the immediate means. In the case before us, the conative process proper operates in resisting or opposing a purely physical or physiological tendency: it might be compared with the effort of a man to open an umbrella in a strong wind. My purpose in the foregoing analysis is to distinguish this type of case, where conation operates in opposition to a merely physiological tendency, from that in which one conative tendency conflicts with another. These two cases are apt to be fatally confused; for they agree superficially not only in the point that one tendency operates in opposition to another, but also in the further point that the struggle between the operations of the two tendencies is normally accompanied by an additional psychical factor of feeling—either mere discomfort, or a highly intense feeling amounting to pain: e.g. any moderate degree of discomfort which may precede the actual sneeze is considerably heightened in the process of endeavouring to suppress it.

A case which resembles more closely still the voluntary attempt to inhibit a reflex process is the stimulation of an emotion; but here the parts played by the physical and psychical factors are reversed. Thus when cognition of the circumstances has aroused conation, and this has naturally developed into specifically directed purpose, there occurs what is termed an emotional experience if this directed purpose is accompanied by irrelevant or perturbing organic processes, which may be assumed to be partially reflex. Thus, in the case of emotion, the physiological causality is apt to interfere with the psychical causality manifested in the form of purpose; whereas in the case of the sneeze it is the purpose which interferes with the reflex process.

§ 9. We may now turn our attention to the highly complex processes of conative conflict, and for the purpose of this analysis I shall introduce a simple mode of symbolism. It may be taken to be the general case that when there are two alternatives either of which can be actualised, there will be in each alternative some aspect or circumstance which is attractive, and some other aspect or circumstance which is repellent. If the aspects felt as attractive be represented by the symbols a and b, and those felt as repellent by the symbols a' and b', then ab' will stand for one alternative, and $a'b$ for the other; and the conflict will be symbolised:

$$ab' \text{ versus } a'b.$$

It is of psychological importance to regard the two symbols a and a' as positively opposed modes of the same generic or determinable aspect; because the mere negative *non-a* could not be represented in thought or imagery with a felt repulsion or attraction; what arouses conation must have some positive content. Only in the limiting case, therefore, where the felt conation was indifferent and could be measured as zero, could we apply the symbol a' or b' to denote merely *non-a* or *non-b*. In order to measure the felt conation we will use the Greek letters corresponding to the Latin, with the sign (+) to stand for the aspect felt as attractive, and the sign (−) for the aspect felt as repellent. Then, if the symbol c stands for the felt intensity of the conation,

$$c \text{ of } a = +\alpha\,; \qquad c \text{ of } a' = -\alpha'\,;$$
$$c \text{ of } b = +\beta\,; \qquad c \text{ of } b' = -\beta'.$$

The resultant conation of each of the alternatives is determined jointly by its attractive and repellent aspects: it may be shortly symbolised as rc. We thus reach the results:

$$rc \text{ of } a \text{ with } b' = \alpha - \beta'\,; \quad rc \text{ of } b \text{ with } a' = \beta - \alpha'.$$

Hence:

$$\text{acc. as } \alpha - \beta' > \text{ or } < \beta - \alpha',$$
$$\text{i.e. acc. as } \alpha + \alpha' > \text{ or } < \beta + \beta',$$
$$\text{so is } rc \text{ of } ab' > \text{ or } < rc \text{ of } a'b.$$

We shall of course assume that the stronger resultant conation is that which wins in the conflict. Now there are occasions when the subject is indifferent about the issue of two conflicting conations. For example, if I am in doubt which to choose of two possible entertainments, say a concert or a game of bridge, then I may be deliberating merely in the sense that I allow the attractive and repellent aspects of the two alternatives, as represented in imagination, to work themselves out, without reference to consequences or other considerations. Deliberative process of this kind, in which the inner working of conative tendencies is passively watched, may be called non-moral deliberation. When, on the other hand, the subject is not indifferent as to which of the conations will prevail, he would seem to have the power to decide the issue. The desire, or conative tendency which may be said to urge him to exert this power, and thus to modify the strength of his primary desires or conations, may be called a *secondary* (or perhaps moral) desire or conation; and we will now examine the practical means by which this secondary process is exercised. On the supposition that it is the first-mentioned alternative that is to be enforced, it will be seen that what is required is to strengthen the felt attraction to *a* and the felt repulsion to *a′*, and at the same time to weaken the felt attraction to *b* and the felt repulsion to *b′*. Thus it is not only the attraction to *a*, but also the repulsion to its alternative *a′* which must be felt more strongly, if the will is to accept *a* and reject *a′*; while if *b* is to be rejected in spite of its attractiveness, and *b′* to be accepted notwithstanding its objectionableness, then the felt force of these factors must be weakened. I attach the highest importance to this double reference to the repulsive as well as to the attractive tendencies; and on this point it appears to me that James—whose symbolic representation of the process agrees to some extent with mine—wrongly represents the matter. I agree with him that an act of attention, which renders more vivid the imagery or more determinate the idea connected with the various aspects of the two alternatives, is the essential factor in the process. But in my opinion, James misrepresents the case when he assumes that the single effect of rendering to oneself an aspect more vividly is to strengthen the inclination to actualise that aspect. This can surely only be the case when the aspect strengthened by attention is itself attractive; for if it is repulsive, then the effect of attention is to strengthen the felt repulsion. In point of fact, in many cases of moral conflict, it is the more determinate thought of what will in some sense pain us in the alternative which our moral attitude directs us to reject, that helps us to decide; rather than the more determinate thought of what will please us in the alternative which the moral attitude prompts us to accept. In the general case we shall include both factors.

Now this power of modifying primary desires and aversions by direct attention is one of the most conspicuous forms in which the ordinary man claims freedom of the will. Since the mere uninfluenced force of his desires and aversions does not inevitably determine the issue and, by exerting the power he possesses over the direction of his attention, he is able to influence the ultimate decision, he holds that therefore there is free will. This very important sense in which freedom of the will has to be maintained does not infringe the principle of causal determination which we attribute to volitional processes: it is not a mere accident or matter of chance whether the secondary desire does arise with a strength sufficient to change the issue; the agent's secondary desires will have had a set of antecedent causal conditions similar to those which we ascribe to the primary desires. Tracing this causal chain backwards, we have of course to presume potential conative tendencies present at the early periods of the developing experience; and science is here, as in other branches of psychology, supremely ignorant, the actual causal conditions constituting a problem for investigation.

§ 10. In this analysis I have referred exclusively to the conative aspect of deliberative volitional processes; but logically speaking, it would have been perhaps more correct to define the cognitive processes before analysing the conative; for the judgment or knowledge which is presupposed when we speak of the alternatives within the range of possible actualisation by the agent, is totally independent of the conative aspects. It is only after judgment with respect to the possible ends has been so to speak impartially exercised, that the conative forces moving in one or the other direction, begin to operate. While, therefore, the cognitive processes without the conative would be inadequate as determinants of the will, just as would the conative without the cognitive, it is nevertheless true to say that the forms assumed, and the prior conditions which account for the forms assumed by the judgment, are totally and absolutely independent of the conative processes. Psychologists who have given a clear enough analysis of cognitive deliberation have made, what appears to me to be, the fatal mistake of attempting to reduce conative deliberation to the same type of formula. The cognitive aspect of a deliberative process is concerned merely with the known or accepted causal conditions for actualising any supposed end, and this process of judgment has the same conative impartiality as any scientific problem—theoretic or practical. That this cognitive or intellective process is to be assumed, seems to me to raise no controversy. I have therefore laid emphasis upon the conative aspect of deliberation since I hold that it is to this process that causality essentially applies. In this view I am opposed to those psychologists who maintain that the will is free in the sense that it can act on mere judgment without any conative urging; and it is upon this issue that the burning controversial problem essentially depends.

§ 11. The above general reference to judgment as essential in the higher volitional processes must be supplemented by a consideration of the different kinds of predicates and their correlated subjects which together constitute the various types of propositional content. So far the judgments entering into the process of deliberation to which we have alluded have been those directed to physical or at any rate external conditions, predicating of these, characters quite independent of mental reference. But the judgment which distinguishes the higher human volitions attributes *value* to possible existents, and in certain relevant cases comparative values to different alternatives; such judgments predicating of their objects characters which are intrinsic to them, in the sense that they are entirely independent of the likes and dislikes of the person judging. Without entering into controvertible issues, it will be universally admitted that when objects are characterised by such adjectives as good or beautiful, they can properly be said to be raised into a realm of reality removed from that realm in which reference is made merely to predicates based upon qualities of sensation, or upon the scientifically developed properties of continuants. At any rate these adjectives 'good' and 'beautiful' are imposed upon their objects in an act which is quite other than the analytico-descriptive characterisations made by what we may call science; and apart altogether from any influence upon volition, this species of judgment has unique features, which distinguish it from the type of judgment with which the simpler logic is mainly concerned. With regard to the influence of such judgments upon conation, it may be that an attitude is necessarily evoked which tends to stimulate the thinker to produce so far as possible the kind of object to which value is attached in his judgment. If so, a judgment of value of this kind may be said to be by itself the sufficient cause of a direct act of will. Where, on the other hand, the judgment is not accompanied by a felt urgency sufficiently strong to overcome conflicting tendencies, it may still be a pure judgment of value. What, in my opinion, constitutes the importance of judgments of this kind, is that where any causal relation between the judgment and the conation subsists, it is the character predicated which causes the conation, and not conversely, the conation or felt tendency to actualise the object, which causes the judgment of value.

CHAPTER IX

TRANSEUNT AND IMMANENT CAUSALITY

§ 1. To understand the distinction between transeunt and immanent causality it is necessary to have grasped the conception of the continuant; and to illustrate how the continuant functions in this connection, it will be convenient once again to analyse what is meant in physical science by movement. We may speak for instance of points A and B being occupied at one instant of time, and the points C and D unoccupied; while at a subsequent instant, points C and D are occupied. In the temporal interval from one instant to another something physical has happened to which the name movement is given. But such movement cannot be unequivocally described unless we are able to distinguish between two such cases as first a movement from A to C and from B to D, and second a movement from A to D and from B to C. Unless we know which of these alternatives is the correct description, our conception of what has happened in the time-interval is undefined, and no subsequent events can be inferred without presuming one or other of these alternatives; so that, in constituting the event called movement, we must assume *something* that moves, to which the name particle may here be given. The one alternative then is that a particle has moved from A to C while another particle has moved from B to D; and the opposite alternative that a particle has moved from

A to *D* while another particle has moved from *B* to *C*. And when we speak of two particles we conceive of each as continuing to exist and as retaining its identity with itself and its diversity from the other; so that terms such as this, or that, or it, involving reference to the same particle, are required to describe what we hold to be the character of the event.

§ 2. From this elementary illustration of a continuant, we may pass directly to an illustration of immanent causality. Thus, when the movement of a particle from *A* to *B* during an interval of time is followed by a movement of the same particle from *B* to *Y*, the law or formula in accordance with which the nature of the former movement determines that of the latter exhibits immanent causality; i.e. the causality in which the cause occurrence and the effect occurrence are attributed to the same continuant. The law in this case is known as the first law of motion, and it can be briefly expressed thus: the speed and direction of the movement of a particle is maintained unchanged from one period of time to another. The empirical establishment of this formula presupposes that no other form of causality intervenes. But when, in the physical domain, one particle is regarded as causal agent and the other as patient, in the sense that the movement of the latter is conceived as the effect occurrence, while the position of the former relatively to it constitutes the cause occurrence, a different notion of causality is introduced, and this we shall call transeunt. Here then the cause occurrence and the effect occurrence are referred to different continuants, whereas in immanent causality cause occurrence and effect occurrence are attributed to the same continuant.

This illustration serves further to indicate what may be assumed to be universally applicable, that any concretely described causal process must be analysed into a conjunction of transeunt and immanent causality; and neither types of causality are to be found actually separate.

§ 3. We now pass to a somewhat complicated physical illustration; namely the case of a gas of which the pressure, volume and temperature are conceived as its alterable states. The gaseous body to which we here refer is not a simple or ultimate physical continuant such as a particle, but consists of an indefinite number of ultimate constituents to each of which the name molecule is familiarly attached. The spatial relations amongst these molecular constituents are alterable, so that the gaseous body as a whole may be said to have an inside, and the terms pressure, volume and temperature are therefore permissible as defining its alterable states. Before we proceed to consider some actual process of experimentation upon such a gaseous body, it may be pointed out that experiment itself implies transeunt causality; for the experimenter employs physical agents whose movements he himself controls, and these produce in the material operated upon, effects which would not have been produced apart from this external manipulation. If, in the simplest case of the laboratory, the bodies there occupying space—being conceived in their combination as a whole—were left to themselves, then the changes which would take place would come under the head of immanent causality. But when the experimenter, by manipulating other bodies which he can control, produces effects which modify the course of immanent causality, these must be described as transeunt.

Let us first consider the case where the pressure upon the gas remains constant, and the experimenter, by means of controlling instruments, alters the temperature and awaits the effect, exhibited as a change of volume, in the gaseous body. Here the mathematician briefly expresses the formula of causality in the equation $pv = kt$, where p, v, and t represent respectively the pressure, volume and temperature of the gas, and k is a constant coefficient which measures a certain unchanged property specific to the gas under experimentation[1]. This familiar mathematical formula is inadequate, however, to express the joint transeunt and immanent causality which we propose to investigate. I shall therefore symbolise two pressures, temperatures and volumes, one applying to the gaseous body itself—which I shall call internal—and the other, which I shall call external, to the surrounding body or envelope, the suffixes i and e being employed thus p_i, p_e, v_i, v_e, t_i, t_e, to represent one and the other continuant. The case before us is that in which gas is contained in a cylinder, the volume of which is free to change as the weight placed on the top changes. The temperature of the gas t_i is then determined by the temperature t_e of the cylinder, which may therefore be regarded as agent in the process described as the conduction of heat which causes t_i to equal t_e. Here, then, t_e being referred to the surrounding body as cause, and t_i to the gaseous body as effect, we have transeunt causality. Similarly the weight of the piston, of which the experimenter has direct control, represented by the pressure p_e, determines the pressure p_i in accordance with the process described as the transference of pressure which causes p_i to equal p_e. Here, then, there are two separately conjoined transeunt causal factors p_e and t_e, each of which entails as a separate effect p_i and t_i. Next the conjunction of the two factors p_i and t_i characterising the gas itself, causally determines the effect v_i, in accordance with the nature of the gas, thus exhibiting immanent causality. Lastly v_i, which is the volume of the gas, determines as its effect the volume occupied by the cylinder which will be v_e, since the expansive property of gas causes v_e to equal v_i. In other words, this last causal process is again transeunt, but it is from the gas as agent to the cylindrical envelope as patient. The whole process, then, may be schematised as follows:

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[1] See Part II, Chapter V, § 9.

$$\begin{array}{ccccc} \text{Transeunt} & & \text{Immanent} & & \text{Transeunt} \\ \left.\begin{array}{c} t_e \rightarrow t_i \\ p_e \rightarrow p_i \end{array}\right\} & \rightarrow & v_i & \rightarrow & v_e \end{array}$$

under the formula (say) $v_i = \frac{kt_i}{p_i}$ (immanent),

and $t_i = t_e$; $p_i = p_e$; $v_e = v_i$ (transeunt).

§ 4. A new problem is at once suggested by the above immanent formula, which connects together the alterable volume, temperature and pressure of the gas, showing them to be related independently of any action upon the gas from without. In Part II under the heading of functional deduction, the notion of the reversibility of cause and effect was treated in its mathematical or deductive aspect. This same principle is illustrated in the case before us by taking indifferently t and p as independent of one another, and v as dependent jointly upon them; or t and v as independent of one another, and p as dependent jointly upon them; and we have now to reconsider the principle under its inductive and causal aspect. In the above account of the experiment, the volume of the gas is represented as effect, and its temperature and pressure as the two cause-factors; but the problem arises, since the factors p and v are manifested simultaneously, how to determine which of them is to be called cause and which effect; and indeed philosophical criticism of the conception of causality frequently suggests the view that where the cause occurrence and the effect occurrence are represented as simultaneous, there is no principle for deciding which of the two occurrences constitutes cause and which effect. Now the general principle whereby I distinguish the cause from the effect where manifestations are temporally coincident, is developed from the distinction and connection between immanent and transeunt causality. To explain this point, let us turn from the above described experiment of a cylinder which is allowed to increase in volume by a movable piston, to one in which the experimenter encloses the gas in an inextensible envelope. The two experiments will agree in respect of the temperature process, i.e. in either case the surroundings are at a certain temperature t_e which, through conduction, will produce a temperature t_i of the gas equal to t_e. But whereas in the previous experiment, the change in volume v_i, produced by the increased temperature, causally determined v_e, in the new experiment, where the volume of the receptacle is controlled by the experimenter, v_e causally determines v_i. The whole process in this case may be schematised just as in the other, by merely interchanging p and v, as follows:

$$\begin{array}{ccccc} \text{Transeunt} & & \text{Immanent} & & \text{Transeunt} \\ \left.\begin{array}{l} t_e \rightarrow t_i \\ v_e \rightarrow v_i \end{array}\right\} & \rightarrow & p_i & \rightarrow & p_e \end{array}$$

Now in comparing the two schematisations from the point of view of the causal process to be explained, it may be asked why, while t_i constitutes a cause-factor in both cases, yet in the first case p_i is said to function as the other cause-factor and v_i as the effect, in the second case v_i is said to function as the other cause-factor and p_i as the effect. Why is it that in one case p_i to v_i stands as effect to cause, and in the other case as cause to effect? In the first experiment v_i was not the cause of p_i, because p_e was the cause of p_i; in the second experiment p_i was not the cause of v_i, because v_e was the cause of v_i. So long as we are only concerned with the alterable states of the one continuant—i.e. so long as we are concerned only with immanent causality—there is absolutely nothing to determine which among the co-variable states is to be called cause and which effect. But as soon as we refer to the surrounding influences, and thereby take into consideration transeunt causality, then that state of the gas which is the immediate effect of the state of the surroundings stands as cause relatively to the other inner states of the gas. Briefly we may repeat the schemes omitting the points in which they agree: in the first experiment p_e causes p_i, and therefore it is p_i that causes v_i; in the second experiment v_e causes v_i, and therefore it is v_i that causes p_i. This is generalised in the following principle: when the co-variable states of a body are causally determined in accordance with an immanent formula, then those variables which are separately effect-factors in the transeunt process must constitute the cause-factors jointly in the immanent process.

It is necessary here to point out that a totally different account of the transeunt and immanent processes which the gas undergoes is required when (in accordance with the kinetic theory) we conceive temperature as molecular kinetic energy, etc. For, here, the free linear movement of each molecule illustrates the process *immanent* to that molecule; and, at the instant of its collision with another molecule, is illustrated an inter-molecular *transeunt* process. Moreover, in this analysis, the events are conceived as successive and not simultaneous. This more ultimate account, however, in no way impairs the validity of the above.

§ 5. It may be helpful to pass from this purely physical illustration to a case of psycho-physical causality which has been much discussed in recent times; viz. as to the relation of an emotion to its so-called expression. Before the James-Lange theory was propounded, emotion was conceived merely as a mode of feeling determined essentially by the cognition of a situation as such or such. This analysis disregarded the perturbing concomitants of such experiences as those of fear and anger; thus while in two such contrasted experiences, the nature of the situation was held to be an object of differentiated cognition on the part of the experient—a differentiation which accounted for the corresponding difference in the purposive acts which ensued—yet in the analysis of the two experiences, no place was given to perturbing organic processes, which were regarded as a mere bye-product of the emotional state. Now James held, in my opinion correctly, that the apprehension of a situation of danger which leads merely to adaptive purposive action, does not constitute a state of fear; nor would a situation in which an experient judged himself to have been injured or insulted constitute a feeling of anger, if it simply led to correspondingly purposive action. A mere cool intellectual judgment—which is not altogether a human impossibility—could not properly be called an emotional state; in order to constitute an emotion, the presence of perturbing organic processes which produce corresponding organic sensations is an essential—or even in James' view apparently *the* essential—condition for the emotional consciousness. In my opinion the only error in James' view is that, while it is true that the cognition of the situation without accompanying organic disturbances would not constitute an emotional state, yet neither would the organic processes constitute an emotion apart from some corresponding form of cognition of the situation. Emotion requires the presence of both cognitive and sensational factors.

Agreeing so far with James' account of the concomitant factors concerned in an emotional experience, we proceed to consider how the various elements are causally related. If it be admitted that the immediate initiative of an emotional experience is the cognition of a situation as being of a certain kind, then this cognition partially illustrates transeunt causality, inasmuch as something presented to perception or imagination occasions the content and form of the cognition. The cognition to which a separate cause has thus been assigned, must be assumed to arouse a conative process, for on no other hypothesis, I think, can emotion be explained; and this conative process, being a manifestation of the nature of the experient, illustrates immanent causality. At this point we again assume a transeunt psycho-physical process, but in the reverse order—viz. from the conative tendency as cause to the physiological process as effect. Thus the conative tendency being aroused suddenly or with intensity, the immediate consequent physiological effects are vaguely, widely, and intensively diffused, in such wise that the physiological disturbances accompanying vastly different kinds of cognised situations have many factors in common. These emotive effects are to be sharply contrasted, in my view, with the purposive effects arising out of the thought element involved in the conation; although this purpose, like the emotive disturbances, constitutes a psychical cause of a physical effect. In this analysis, the conation as a psychical process is seen to be the effect of an external cause, and in its turn, the cause of whatever specific occurrence may thereafter take place. We shall not say, therefore, after the old fashion, either that the emotion of fear causes the physiological disturbances, or that the physiological disturbances cause the emotion; for the emotion is not simple, but a compound of cognitive, conative and sensational factors.

§ 6. Having illustrated how the notions of transeunt and immanent causality are employed in physics, we will now consider the much more complicated case of psychology, assuming the philosophical position known as dualism, which regards the psychical continuant as something with a nature fundamentally distinct from that of a physical continuant. Just as the one indubitable illustration of a physical continuant is the particle, so I shall assume that the only indisputable psychical continuant is an experient, or (what for the present I wish to take as synonymous) a person, a mind, a self, or an ego. Now the person and the particle agree in being what I call a continuant, namely something which continues to exist and to stand in relation to what changes in the course of time. The continuant, in either case, must be distinguished from its property, since the property may either remain unchanged, or may change within a given period of time; the terms unchanging and changing, therefore, apply to the properties (as well as to states and relations), and not to the continuant itself. Of the particle, physicists maintain that it has one property, namely mass, which is unchanged; and the abstract dynamic theory, which has held the field almost uninterruptedly *since Newton, is that mass, besides being the only unchanged property of a particle, is the *only* property that can be attributed to a particle as such. Whether ultimately all particles have the same mass; or whether different particles have different masses, is irrelevant to our present purpose; and I shall assume provisionally that in either case mass is a property of the particle, which is to be distinguished from any complex quantity definable in terms of motion and in particular of acceleration*.

Contrasting the notion of a particle with that of the psychical continuant or person, we may say of properties attributable to any given person in the first place, that they will not agree determinately with the properties of any other person; and that in the second place, they are subject to change in the course of time. Thus the concept of a psychical continuant differs from the outset in two important respects from that of the physical continuant, in that (1) no property of a psychical continuant is determinately the same as that of another; and (2) that no determinate property attributed to a psychical continuant remains unchanged during an indefinite period of time.

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* The modifications of Newtonian physics at the present day strengthen—rather than weaken—the form of my logical analysis: the instructed reader may easily make the requisite corrections.

But a far more important and far reaching distinction between the two, is that a psychical continuant may be said, metaphorically speaking, to have an *inside*, while the physical continuant has not. In other words, apart from the property of mass, nothing can be predicated at any time of a particle except its position relatively to other particles, and the change in its relative position as time lapses; * while we may speak properly of the complex analysable *states* of the psychical continuant. Thus the term change, when applied to a particle, refers solely to the external relation, position*; but when applied to the psychical continuant it refers predominantly to alteration of state. It is true that science speaks of the state of a material body, where the body is conceived as containing a number of particles; but when the term 'temperature,' for example, is used to denote such an alterable state of a body, it means nothing more than the mode of agitation of the particles which constitute the body. Since this mode of agitation (i.e. of relative movement of the particles within what in many cases continues to be the same body) is subject to change, the conception of an alterable state of a material body is legitimately applicable to its temperature. Other conceptions have been introduced into physical science, resembling temperature in these respects; i.e.

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* This again requires modification in the light of present-day physical science.

they represent alterable states of a body, logically distinguishable both from the aggregate of particles which constitute the body itself, and from the properties which can be attributed to the body as distinct from the mass attributed to each of the particles of which it is composed.

The property of a composite body—i.e. a body composed of an aggregate of particles—is to be defined, not as anything actually manifested in time and space, but as a rule in accordance with which the changes actually manifested in time and space can be formulated. Material bodies may be grouped according as they continue to be composed of the same particles—such bodies being called inorganic—or according as they are composed of minor parts differing from time to time, these being assimilated from external bodies, and uniting in modes regulated by a *property* of the body, i.e. a rule of behaviour formulating the processes of these minor parts. Such bodies are called organic.

§ 7. The further consideration of the character of psychical causality leads to the introduction of at least two fundamental factors *uniquely* characteristic of mind, although a tendency has prevailed throughout the history of philosophy, to import into interphysical relations these conceptions which are true with certainty and immediateness only of psychical processes. In the first place to mind or consciousness or experience, I attribute efficient causality—a notion which can have but dubious application in the physical sphere, while there can be no doubt that something equivalent to power is a mental factor which can and does influence the course of physical events. And secondly, there are the processes of cognition, judging or thinking, which are of still more importance both as causally determinative, and because, regarded as effects manifested as occurrences in time, they come under a special kind of ontological determinism. Volition, which is a psychical act, determined jointly by conative and cognitive processes, exhibits both these peculiar characteristics of mind, and it is directly determinative of physical occurrences. From the time of Aristotle, the second unique aspect of mental process mentioned above has been described as final cause; and here, as elsewhere, I hold that a strictly factual account, and not a merely metaphysical explanation, is to be given of this notion. In final causality, the idea of E, an effect or end, is an essential causal factor in the actualisation of E; but at the same time, the whole significance of the conception of final cause is that it is in itself an efficient cause. When an occurrence is explained in terms of the end which a conscious and thinking being has in view, the end or final cause would appear to be functioning as efficient cause; but it must be borne in mind that the mere idea of an end can only constitute an efficient cause of the actualisation of that end in so far as it involves an act of will which, in my analysis, constitutes a crucial occurrence within the psychical processes which take place in the course of time.

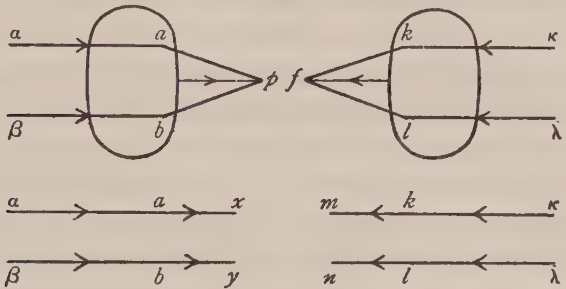
§ 8. Up to this point I have sketched the logical conceptions employed in a general philosophical account of psychical processes, and have therefore only raised problems open to philosophical as distinct from strictly psychological controversy. A specifically psychological explanation of the way in which feeling and desire enter along with cognition or thought as jointly determining the critical act called volition, is given elsewhere in this volume; and at this point I have merely put forward a brief analysis of an act of volition as guided by cognition and motived by conation, as the most important illustration of purely immanent causality.

In further explication of the notions of transeunt and immanent causality in science, it must be pointed out that when the action of one continuant x upon another continuant y exhibits transeunt causality, the mode in which the states of y are thus causally determined cannot be regarded as dependent *merely* upon its relation to x, since conjointly with this relation, the mode depends upon some character peculiar to y itself. If this specific character of y be called immanent, we have an illustration of the way in which along with transeunt causality an immanent factor enters. But, when we conceive of a plurality of continuants as in some way constituting a single continuant, then, although certain causal processes are correctly conceived as immanent relatively to this unitary whole, yet they must be conceived as transeunt relatively to the several constituents. The transeunt interactions between the several constituents may or may not be known at certain stages of scientific development, and when unknown we are limited to conceiving the processes as immanent within the composite whole, while the further theoretically possible knowledge which would resolve the immanent causality into transeunt processes amongst the constituents, would not upset our previous application of immanency, but would represent a more penetrating and ultimate knowledge of the facts. An example of this alternate way of representing causal processes has already been given. Perhaps the principle called the conservation of energy would illustrate the matter as well as any other. This principle conceives of a system of particles or bodies acting on one another, sometimes in highly complicated modes, preserving throughout all its changes a constant quantum called energy; and thus the formula of conservation of energy represents immanent causality, inasmuch as it regards the system as a whole, and as not transeuntly operated upon from without. At the same time, detailed knowledge of the forms of energy which *change* in the causal process, may be conceived under transeunt causality as operating amongst the particles of the system. This illustration from physics may be compared with the economic conception of a society of persons. Thus the formula according to which prices of commodities are maintained unchanged so far as the community is not transeuntly operated upon by other communities, is analogous to the conservation of energy; but the further analysis of the processes of exchange and contract between person and person presents the facts more ultimately and more exactly as involving transeunt relations between the persons; just as in physics processes immanent to the whole are more profoundly defined in terms of transeunt causality as regards its parts.

CHAPTER X

CONVERGENT AND DIVERGENT CAUSALITY

§ 1. The whole topic of causal determination may be approached from a different point of view by considering the complex relations of interdependence amongst factors of events such as the terms cause and effect are familiarly used to describe. It will simplify the exposition of this aspect of the problem to introduce a little elementary symbolism, and throughout this chapter the reader is asked to bear in mind the following diagrams:



If both a change in a alone and a change in b alone would entail or point to a change in p, where a and b are cause-characters and p an effect-character, then we shall speak of the *convergence* of the cause-characters a and b towards p. In the same way, if both a change in k alone and a change in l alone would entail or point to a change in f, where k and l are considered as effect-characters and f a cause-character, then we shall speak of the *divergence* of the effect-characters k and l from f.

Now, when *a* and *b* converge towards *p*, it will also be the case with respect to effect-characters other than *p* —say *x* and *y*—that a change in *a* alone would entail a change in *x* without a change in *y*, and a change in *b* alone would entail a change in *y* without a change in *x*. Processes such as these, from *a* to *x* and from *b* to *y*, may be said to be parallel to one another. In the same way, when the processes from *f* to *k* and *f* to *l* are diverging, it will also be the case with respect to cause-characters other than *f*—say *m* and *n*—that a change in *k* alone would point to a change in *m* without a change in *n*, and a change in *l* alone would point to a change in *n* without a change in *m*. Again then, processes such as that from *m* to *k* and from *n* to *l* may be said to be *parallel* to one another. The additional characters $\alpha, \beta, \kappa, \lambda$ are introduced in the above diagrams in order to exhibit more fully the significance of parallel, converging and diverging processes.

In these diagrams the horizontal lines from left to right represent the course of time from before to after, so that any vertical line that may be imagined represents simultaneity. On the other hand, the arrows, which are sometimes directed rightwards and sometimes leftwards, indicate—not the temporal opposition of before and after—but the inferential opposition between implying and implied. In speaking of the temporal process from *ab* to *p* as converging, we mean that when *a* and *b* are jointly manifested, they both play a part in determining for us what value *p* of *P* will be manifested. This converging process is represented as preceded by parallel processes αa and βb, while it will be observed that the parallel processes *ax* and *by* are contemporaneous with the converging process from *ab* to *p*. Similarly in speaking of the temporal process from *f* to *kl* as diverging, we mean that when *k* and *l* are jointly manifested, they both play a part in determining for us the value *f* of *F*. The diverging process is succeeded by parallel processes *kκ*, *lλ*, while again the parallel processes *mk*, *nl*, are contemporaneous with the diverging processes to *kl* from *f*. In the interpretation of these diagrams, it is important not to be misled by the irrelevant spatial relations which inevitably enter in diagrammatic representations; for example, the relations of parallelism might falsely suggest such kinematic notions as two particles moving contemporaneously in parallel directions from the points α, β, to the points *a* and *b*, and then converging towards the point *p*. A special application of these diagrams might of course be made to such a kinematic problem; but if it were, the full significance of the diagram could only be exhibited by representing the two particles, whose courses are α*a* and β*b* respectively, not as moving in parallel lines, but as colliding at the moment represented by *a* or *b*, this collision altering the direction of the particles and accounting for the state of things at the moment represented by *p*, which closely follows the moment *a* or *b*. The parallelism of the lines in the diagram represents the causal or determinative independence of the movements of the particles prior to their collision, and not their spatial directions; for in fact they must have been spatially converging in order to lead to the collision. Another possible interpretation of the metaphor of parallelism which it is important to avoid in the use of these diagrams is that which applies to the case of psycho-physiological parallelism. There a knowledge of the physiological processes enables us to infer the contemporaneous psychical processes, and conversely; so that parallelism, meaning here epistemic determination, would be represented by two oppositely directed arrows. But in the case which the diagram represents, and which might be illustrated by two billiard balls, the motion of one ball before collision with the other would inferentially determine *no* knowledge whatever of the preceding velocity or direction of the other. Parallelists who tried to combine the notions of determinative dependence and causal independence would have to revise the entire logical account of causality and its connection with epistemic determination.

§ 2. It will give additional significance to our diagrams to further elaborate the kinematic problem. The straight part *aa* of the crooked line *aap*, might stand for a movement of uniform velocity and constant direction, or rather for the constant retardation due say to the friction etc. of a ball moving on a billiard table. Then the change from *aa* to *ap* would represent the change of acceleration or retardation which takes place at the moment of geometrical contact with a second ball whose previous course is represented by *βb* and whose changed acceleration is shown by *bp*. Furthermore the junction of *p* with *f* might represent the process of contraction and re-expansion due to the elasticities of the balls. And, lastly, *kκ* and *lλ* would exhibit their subsequent rectilinear and causally independent movements.

To show that these diagrams are adaptable to very different kinds of phenomena, let us now turn to chemistry, and suppose *a* to represent the continuously manifested properties of a sample of oxygen, and *b* those of a sample of hydrogen. These properties would continue to be manifested without change until some environmental condition brought the two substances into a special spatial and physical connection. If *a* and *b* represent the moment at which this connection takes place, then at the moment *ab* there is initiated a process of chemical combination which leads to the manifestation of new properties—viz. the properties of water—symbolised by *p*. The arrow from *ab* to *p* represents the fact that our knowledge that oxygen and hydrogen are the elements concerned determines for us the knowledge that the properties of water will be manifested in the combination; these properties being jointly determined by the definition of the elements and their spatial relations. As, in the kinematic illustration, so here an appearance of discontinuity or abruptness is presented at the moment when the oxygen and hydrogen combine, and are replaced by the properties of water. Further *ax* and *by* may be taken to represent the continuance unchanged of the weights of the oxygen and the hydrogen that are combining; these unchanged and independent values continuing to be manifested contemporaneously with the process which ends in the chemical combination. In this way the illustration from chemistry affords an example of the universal principle that along with any converging process of causality, there are always contemporaneous parallel processes—the words parallel and converging being used metaphorically to stand respectively for causal independence and joint dependence. By bringing into line these examples from chemistry and dynamics we have partially shown in what respects chemical combination and dynamical composition agree, and in what respects they differ. The complete change of motion of the two colliding balls corresponds to the change from the manifestation of the chemical properties of oxygen and hydrogen to that of the properties of water. And the continuance of the weights of the oxygen and hydrogen unchanged during the process of combination corresponds to our conception of the continuance of the masses and resultant momenta of the two balls during the process of collision.

§ 3. The same diagrams serve to illustrate the causal connections between sensation and physical stimulus. We will suppose that the moment *a* or *b* is that at which the subject becomes aware of two sensations—say of the colours red and blue present simultaneously apart in the field of vision. The physical processes preceding these sensations are represented by *αa* and *βb*, parallelism of the lines standing for the presumed causal independence of these two physical processes. Now, when the sensations *a* and *b* happen to be incidents in the experience of the same percipient, there will be consequences which would not be entailed if these sensations were experienced by different percipients; these consequences, which we will suppose to be the apprehension of a relation of difference between the two sensations, will be represented by the converging process from *ab* to *p*. The arrow from *ab* to *p* illustrates the fact that the apprehension of the relation of difference will be determined by the apprehensions *ab* jointly. The contemporaneous processes *ax*, *by*, might serve to illustrate the continuance unchanged of the apprehensions *a* and *b*, while the apprehension of their relation of difference is being evolved. But there are many other ways in which the diagram could be interpreted to illustrate the psycho-physical process of sensory stimulation. For instance, if *a* and *b* stood for the neural processes occurring contemporaneously with the sensations, these would continue to pursue their course, in some respects at least, unaffected by the percipient's cognitive processes of comparison and so on; and in this case the more or less parallel processes *ax* and *by* would be contemporaneous with the converging process from *ab* to *p*.

The psycho-physical illustration may be carried further by supposing the points *p* and *f* to be joined up; this enables us to treat a more complex problem. Thus, if *a* and *b* stand for different simultaneous occurrences whose relations or connections are apprehended in a synthetic cognitive act (symbolised by *p*), the nature of this perception is causally determined by the nature of *a* and *b* jointly; hence the arrow passing from *ab* to *p*. Next taking *p* as cause, the effects which follow will be other than those which could have been predicted from a knowledge of the separate processes αa and βb; these latter consequences, which occur independently of *p*, will be symbolised by the lines *ax*, *by*, continued indefinitely in straight or converging directions, of which the course may be said to be parallel to the mental processes and their consequences—the word parallel being used metaphorically to signify determinative independence. For instance, such phenomena as light or heat will engender various physical consequences in the outer world concurrently with the mental processes and purposes of any individual percipient; and these physical consequences will, in most of their aspects, be independent of psychical process until some new converging process, involving what we may call metaphorically another collision between mind and matter, takes place. Meantime, let us consider in further detail the effects following upon p, which represented the convergence of physical causes to a psychical effect, these effects being represented by divergent processes in which the causality is from the psychical to the physical. Let us suppose that the perception p develops, owing to such conditions as the percipient's character and past experience, through processes of cognitive and conative deliberation, into a fiat of the will (symbolised by f). The causal process of inner deliberation is represented by a line which may be supposed to join p to f. Then giving to the effects of f the same kind of complexity that we have attributed to the causes of p, k and l may be taken to represent the diverging manifestations of the fiat f. The arrow pointing from kl to f indicates that the observer can infer from the joint manifestation of k and l the character of the fiat f which caused this manifestation. If then k and l represent those phases in the causal process over which the experient has no longer any direct control, k will develop causally into κ, and l into λ, along independent lines, such that from κ alone we could infer k as its cause, and from λ alone we could infer l as its cause. Thus the two parts of the diagram are joined up, and it is seen how the two independent causal processes αa and βb may issue—through the intermediary processes from p to f—into the two independent causal processes $k\kappa$ and $l\lambda$. Now knowledge of the laws of causal determination according to which α evolves into a, and β into b, would not by itself enable us to derive the subsequent processes κk and $l\lambda$. Though all these four processes be taken to exhibit the laws according to which physical phenomena are regulated, no mere physical law will account for $k\kappa$ and $l\lambda$ as consequences of αa and βb. To explain these physical consequents of physical antecedents, we must interpolate a converging, an internal, and a diverging process of causal determination whose sphere of operation is the inner consciousness of an individual experient. The joint diagram may be shortly said to represent the alternate action of matter upon mind and mind upon matter.

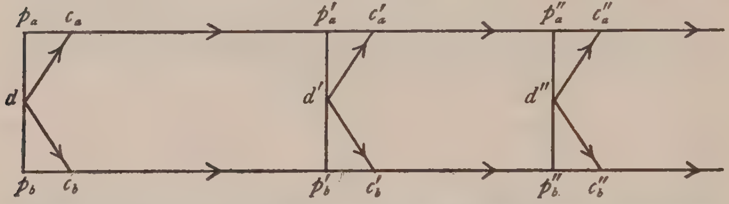
With regard to *inference* in the case of divergent causal process, while the distinct lines fk, fl indicate that the process may be analysed into two or more distinct part processes, the single arrow pointing backwards from kl to f indicates that in general we can infer the determinate value f, only from a knowledge of both k and l, and not from k alone nor from l alone. The symbols may, however, represent an analysis from which f could in certain cases be inferred either from k alone or from l alone. To give an instance of inference from joint factors forwards and backwards, we may pursue the course of two billiard balls, forwards from αa and βb to κk and $l\lambda$, taking p to represent the forces of compression on both the balls, and f the forces of expansion on both the balls; then p would contain the conditions from which we could infer backwards both a and b, and f would contain the conditions from which we could infer forwards both k and l. Equally well we could have inferred forwards from αa and βb the moment and position at which the balls will touch, and from $k\kappa$ and $l\lambda$ we can infer backwards the moment and position at which they have touched. But we cannot infer $k\kappa$ from αa alone, nor αa from $k\kappa$ alone without taking into consideration the movement of the other ball which introduces the converging and diverging processes. Illustrations of the diverging process in which we infer backwards from the conjunction of two or more effects, the nature of the cause, are well furnished by the case of symptoms. Thus in medical diagnosis it is often impossible to infer the nature of a specific disease from any of the symptoms separately, and it is therefore necessary to join different symptoms in order to infer their cause. Similarly the effects of different emotions such as anger and fear, as manifested in bodily disturbances, partially agree and partially differ; hence a number of factors would have to be noted in order to infer in any given case whether the cause of the bodily disturbances was fear or anger. Purposive action affords a peculiarly interesting example of our analysis of causality into converging and diverging processes. Such action may in general be defined as involving a divergent process issuing from a thought of an end, followed by a convergent process in the outer environment in which this same end, previously represented in thought, is actualised in fact.

§ 4. We will now consider certain more complicated cases of causal process which exhibit convergent, divergent and parallel processes contemporaneously. For illustration we will take two particles whose movements are determined under some such law as that of gravity. Let a be one particle, and let p_a and p'_a be two positions successively occupied by a at two moments separated by an assigned interval of time. We cannot infer p'_a from p_a alone, but only from knowledge of p_a jointly with the change of motion which a is undergoing when at position p; if this change of motion be symbolised by c_a, we may then speak of p'_a as determined jointly by p_a and c_a. Adapting our previous diagram to this relation of causality, we have the following:

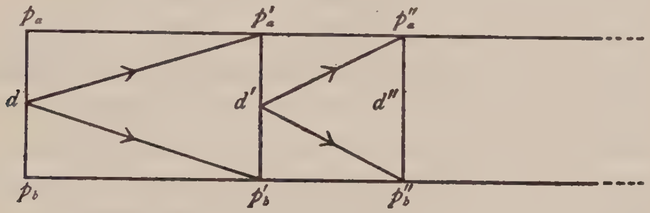
$$\begin{tikzcd} \genfrac{}{}{0pt}{}{p_a}{c_a} \arrow[r, dash] & p'_a \end{tikzcd}$$

where p_a, c_a, p'_a take the place of a, b, p respectively, and the figure represents a converging process. Before introducing a second particle b, we will simplify the above diagram by bringing c_a close up to p_a, understanding by this juxtaposition literal simultaneity, and then join p_a, c_a to p'_a by the horizontal arrow. The motion of particle b is similarly represented by symbols in the second diagram. Now when a is at position p_a, and b is simultaneously at position p_b, the distance between p_a and p_b is a determining condition from which we can infer the change of motion of both a and b, under the law which we have assumed to be that of gravity: this relation of distance, therefore, stands as a cause-condition diverging into the two effects c_a and c_b. The process is exhibited in the following diagram, which also illustrates subsequent positions of the particles subject to the same type of causality. Here, at the first moment, the distance d between p_a and p_b determines divergently the changes of motion c_a and c_b, while the position p_a and the change c_a determine convergently the position

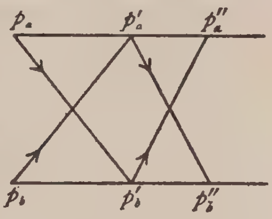
p'_a; similarly the position p_b and the change c_b determine convergently the position p'_b. It is also important to note that the converging process $p_a c_a$ to p'_a is determinatively parallel to, that is independent of, the converging process from $p_b c_b$ to p'_b. The same relations are exhibited at the next moment considered in relation to the third moment, where the dashes serve to distinguish the several moments. Between each of the moments separated in the diagram, we must suppose an indefinite number of the same configurations following one another continuously. And since, at any instance, the distance *d* is a causal factor common to the movements of *a* and of *b*, the movements from p_a to p'_a and from p_b to p'_b are not properly called parallel in the determinative sense, when an appreciable interval of time has elapsed. It must also be remembered that the positions and juxtaposed changes of motion are to be conceived as literally simultaneous and not as continuously successive. We have spoken of p_a and c_a as jointly determining p'_a; we may equally speak of p_a and p'_a as jointly determining c_a; and this illustrates the commutative principle for what has been called a prime dependency. The three values p_a, c_a, p'_a may be said to constitute a kinematic prime dependency: it is actually by the observation of p_a and p'_a that we infer c_a, while we regard p_a and c_a as causally determining p'_a. Thus a knowledge of p_a, p'_a



and p_b, p'_b, could take the place of the knowledge p_a, c_a, and p_b, c_b, in determining the whole course of the action both backwards and forwards. The diagram could then be simplified by omitting the symbols for change of motion on a principle analogous to the triangle of forces, so that a single arrow from d to p'_a will replace the two arrows from d to c_a and c_a to p'_a, thus:



To indicate the continuity of the process, we may still further condense the diagram. In the former of these two diagrams p_a and d determine convergently p'_a; while p_b and d convergently determine p'_b, these two processes being themselves divergent. In the latter of the two diagrams we represent p_a and p_b as convergently determining both p'_a and p'_b, while p'_a and p'_b are divergently determined by p_a and p_b. A slightly different interpretation of the symbols will elucidate this apparent contradiction. If the symbols p, instead of representing the mere geometric notion of position, be interpreted kinetically to include position and determinate motor tendency, the relation d of b to a will then be conceived as a causal condition modifying the motor tendency and thus effecting an actual motion other than that which the tendency by itself would have produced.



The cause conceived in this way has no effect peculiar to itself, but modifies what would otherwise have been the determinate effect. The phenomenon b, when in the relation d to a, which thus modifies the process of a, is conceived as agent relatively to a, and a is conceived as patient relatively to b. In philosophical terminology we speak of the unmodified process of a as illustrating immanent causality, and the modifying influence of b upon a as illustrating transeunt causality. Apart from these disputable terms, the consideration with which we are here concerned is that in order to define the nature of the effect which the relation d of b has upon a, it is necessary to introduce reference to the trend or motor tendency which a is manifesting at the moment when b influences it. This serves to illustrate the point that the idea of change is complex, and needs to be carefully examined: it does not mean simply a difference in the state of a at one moment as compared with its state at a subsequent moment, but it means a difference between the state into which a passes under the operation of an external causal agency, such as its determining relation d to b, as compared with the state into which a would have passed by its own agency.

§ 5. This complex form of causality may be illustrated from psycho-physical process as well as from dynamics. In this case the symbol a in the condensed diagram will stand for the mental side of such a process, and the symbol b for the physical side; the letters p representing not statically defined states, but motor trends. Thus if p_a represents the course which a sensation process is taking at any moment independently of any physical process such as b, the physical stimulus p_b, as soon as it begins to operate, will affect this sensational trend, and determine it in the form p'_a, which is a modification of what p_a would have become *apart* from the stimulus. Now, if the subject is what may be described as *inactive* with regard to the further course of his sensations, the arrow in our diagram will be drawn only from the line of b to the line of a, and the arrows in the opposite direction may be omitted. The diagram would then represent a state in which the sequence of sensations was wholly determined by the course which the physical or physiological processes assume under purely physical laws, and where there was no reaction from the side of the psychical to the side of the physical. But now let us suppose that the subject is active and takes a share in determining the course of his sensations. It must be admitted that such active determination by the subject is not a direct causal determinant; and the facts are illustrated by the diagram with all the arrows inserted. Thus we shall define p_a not as a mere passively received sensation, but as a cognition, having in it an element determined by the nature of the stimulus p_b, and besides this, other related cognitive elements more or less complicated according to the degree of intelligence of the supposed subject. The arrow, therefore, from p_a to p'_b indicates that p_a, defined—not as a mere sensation—but as a cognition, causally determines p'_b in the same way as p_b was previously shown to determine p'_a; that is, p_a does not bring p'_b into existence, but it determines the actual form assumed by the physiological process b, in the sense of modifying the form p_b would have taken, apart from the active determination of this cognition. The whole process is descriptively condensed in the phrase that the physiological course and the sensational course reciprocally determine one another; neither would be what it actually is, if the influence of the other had been non-determinative. This condensed description, however, is more accurately analysed into an alternate process from the side of mentality to the physiological and reversely from the physiological to the side of mentality. The mentality involved is not purely passive sensation, but actively determinative cognition, involving (at least) what psychologists call attention; and in cases of a higher level of intelligence, more or less co-ordinated purpose. The process indeed, which the subject cognises, is itself mental, and must not be confused with the course of the physiological changes themselves, of which he is wholly unaware; his attention is actually directed to the changes in the sensational experience of which he is retrospectively and more or less prospectively aware.

The condensed diagram interpreted so far to apply to the causal interrelations between a merely physiological process on the one side and active mentality on the other, can be used to illustrate a wider range of interaction between mind and matter, which shall include operations on the external environment. In this application the line b no longer represents purely physiological processes, but includes processes in the external physical world. Here again the important consideration is that the purposive thought p_b does not bring into existence the physical phenomenon p'_b, but it determines the phase of b to be otherwise than what p would have become under the determination of purely physical causality. Of course the mode in which the course of p_b passes into the phase p'_b is not adequately represented by a single straight line, and in the diagram a very complicated process is artificially condensed. In fact there will be sections of the physical process that are left—uninfluenced by the subject—to follow a course determined by purely physical causality; and to represent such sections, the arrows from the side of mentality to the physical side should properly be omitted. Changes of this kind may be observed by the subject, and his observation of the phase into which the process has passed may determine him to initiate a new interfering or controlling operation which will again modify the further course of the physical process. The moment when this observation occurs will be marked by an arrow from the physical side to the side of mentality, since it is the nature of the physical occurrence which determines the content of the predicative cognition on the part of the observer. In its turn, this cognition will operate on the other variously modifiable inner processes of the mind, and determine a corresponding reaction, modifying the physiological as well as the physical course of things; and these changes will be marked by an arrow from the side of mentality to that of the external and physical. This arrow is again a condensed representation of converging process; for the phase p'_b determined from the side of mentality, is due jointly to the just preceding observation, taken along with the appreciation of p'_b as a phase in the progressively attained purpose, as well as the knowledge of the activities needed for furthering this attainment.

Finally we may close the exhibition of the entire purposive process by a set of lines converging upon that terminal phase of actualised experience which denotes the realisation of the end corresponding to the thought of the end from which at the beginning the initial processes diverged. Thus, the scheme of purposive causality begins and ends as a phase in the consciousness of the same individual thinker or actor; while the intermediate or instrumental phases are incidents in the world of physical phenomena, some of which are within the organism and nervous system, and thus in the most direct causal contact with the thinker's feelings, thoughts, and powers of causal determination.

CHAPTER XI

TEMPORAL AND SPATIAL RELATIONS INVOLVED IN CAUSALITY

§ 1. THE general discussion of connectional determination entails consideration of the spatio-temporal relations amongst phenomena in terms of which occurrences are represented as bound together in a unity of connection. Thus, the bare formula '*abcde* determines *p*,' where the symbols stand for the characterising adjectives of occurrences, is a merely abstract expression of the causal principle, inasmuch as no reference is explicitly made to the spatio-temporal nexus (as it may be termed) under which the manifestations of these characters take place. When the event characterised as *abcde* is said connectionally to determine an event characterised as *p*, these events have spatio-temporal extension as also spatio-temporal relations one with another. The manifestations *a*, *b*, *c*, *d*, *e* will be termed *occurrents* severally, and their conjoint manifestation will be termed an *event*. These occurrents are *several*, because the determinate character of each comes under its own determinable *A*, *B*, *C*, *D*, *E* respectively. On the other hand, *events* are several, because each has its own distinct spatio-temporal boundary. The *extension* of an event allows us to speak of any event as containing spatio-temporally distinguishable parts, which are themselves events. On the other hand, the occurrents *a*, *b*, *c*, *d*, *e* are not parts but constituents of the event, for they all occupy one and the same spatio-temporal position defined by the boundary of the event. Furthermore, connectional determination signifies that the *position* of the manifestation of *p* relatively to that of *abcde*, as also its determinate *character p*, is determined jointly by the characters of the coincident manifestations *a*, *b*, *c*, *d*, *e*.

So far we have treated the notion of nexus as concerned solely with spatio-temporal relations. But the above account must be amplified and, in a sense, partially amended by noting that every occurrent must be referred to its own proper continuant. Thus, to the contrast between occupying the same or different positions must be added that between being referred to the same or to different continuants. In fact, distinctions of position must be understood metaphorically to extend to distinctions of continuant-reference. And, for similar reasons, determinables must be distinguished according as their determinate values characterise manifestations referred to this or to that continuant.

§ 2. We will now examine the general notion of Order. Order is predicated of terms which, for convenience of figurative representation, may be called Points, and when Points are in an Order the collection is called a Series. Taking any three points whatever in a Series, these may be said to be in a determined order when there is some assignable principle according to which one of the three points is to be regarded as 'between' the other two. Thus the three notions of a Series, or Order, and of 'betweenness,' mutually involve one another. These remarks apply equally to what may be called a discrete Series, or to a continuous Series—two types of Order which may be distinguished as follows: a continuous series is such that between *any* two non-identical Points, there is a Point non-identical with both. The series of integers 1, 2, 3, 4, ..., on the other hand, illustrates a discrete series, for, between the two integers 3 and 4 for instance, there is no integer; again the dots after 4 may also be taken to illustrate a discrete series, for between the first and second there is no dot, as also between the second and third, and so on. A discrete Series is, in fact, always figuratively represented by dots spatially separated along a line; and the fact that it is natural to name these points by the ordinal numbers shows that a series of integers most naturally illustrates a discrete Series. Similarly it is natural figuratively to represent a *continuous* Series by a line, containing points such that between any two points there is in the Series a point different from both. When a line is regarded as the boundary between two contiguous areas of a surface, it enables us to conceive of a *discrete* series of areas; thus we can count one by one a series of contiguous areas by mentally representing the lineal boundary common to any two; but in such case the entire surface is to be described as *continuous*, for between any two lineal boundaries, there is in this surface a lineal boundary different from both. The surface itself may be regarded as the boundary dividing a region into a discrete series of parts; but again in this case, the entire region is to be described as continuous; for between any two areal boundaries within the region, there is an areal boundary different from both. We are thus led to distinguish between the parts of a whole, and the boundaries between these parts. The parts of a line are lines, the parts of an area are areas, the parts of a region are regions; but the boundary between contiguous parts of a line is a point, and the boundary between contiguous parts of an area is a line, and the boundary between contiguous parts of a region is an area or surface. The parts of a whole, therefore, are homogeneous with one another and with the whole; but as we noted when contrasting extensive with extensional wholes[1], the boundaries between two contiguous parts are always of one lower order of dimensions than the parts.

Our illustrations of discreteness and continuity have so far been taken solely from Space; but the notions are equally applicable to Time. Thus Time is conceived as of one dimension, and is composed of parts which are periods, the boundary between two contiguous periods being called an instant. A period of time, therefore, corresponds to a line, and an instant corresponds to a point; the period-parts of a period will then constitute a discrete Series, and the instants—i.e. the boundaries between two contiguous period-parts—will constitute a continuous Series. The above application of the term 'discrete' to contiguous parts of a whole might be criticised as being incompatible with its original application to separated points. But it must be noted that the notion of discreteness does not imply *factual* separation, but only separation *in thought*. When we think of a boundary between two contiguous parts, we are mentally separating those parts, without predicating any factual separation; in this sense we may always say that a whole can be conceived as a sum of its discrete parts, whether the whole is such that it can be said to contain contiguous parts, i.e. parts having a common boundary, or not. In the former case it is usual to call the whole continuous, and unusual to allow of its being also called discrete; but it appears to me that the notion of continuity is derived from that of contiguity, and that the definition of contiguity involves the notion of a common boundary between two parts; hence, for the notion of a continuous whole, I prefer to substitute the notion of a whole consisting of parts, whose boundaries constitute a continuous Series.

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[1] Part II, Chapter VII, § 8.

§ 3. Having contrasted continuity with discreteness, we will now examine another meaning frequently attached to the word continuous, which may, I think, be conveniently contrasted with *discontinuous*. The term discrete, as hitherto explained, applies to a single variable whose variations are not considered in connection with the variations of other variables. We have now to consider so-called *correlated* variables, the variation of one of which is dependent on that of the other according to some assignable formula. In this case we shall find that while the independent variable is continuous, the changes of the other variable correlated with this continuous series may be either continuous, or (as it may be described) discontinuous. The dependent variable will be said to vary continuously when whatever section of its actual variation is considered, every possible value intermediate between those assumed at the beginning and end of the section are actually represented; it will be said to vary discontinuously when within *some* section of its actual variation there are intermediate values which are not represented. The most familiar instance of this kind of correlated discontinuity or continuity are those in which Time is the independent variable. The actual variations of Time are continuous in the first sense of the term, and every possible value is actually represented; but there may be gaps in the variations of the variable which depends, according to some formula, upon the variation of Time. A simple illustration is that of acceleration: thus if a body is moving on a rough horizontal table until it falls over the edge, then at the instant when it begins to fall there is discontinuity in the change of acceleration. While it is moving horizontally its movement is subject to the retardation of friction operating horizontally; when it is falling, on the other hand, its movement is subject to the acceleration of gravity which operates vertically. There must, therefore, be an instant in which the acceleration or retardation changes from one value to another with the omission of all the possible values intermediate between the horizontal retardation and the vertical acceleration. The acceleration in such a case varies discontinuously, but not so the *velocity*; for every possible velocity intermediate between the rate of movement of the body on the table and the rate when it is beginning to fall, is assumed by the body during the intermediate time; for the body does not fall vertically, but—neglecting the resistance of the air—along a parabola.

Now according to a prevalent view in philosophy, the theory that *all change is continuous* is intuitively axiomatic. But the change of acceleration in the above case would seem to contradict this theory; although physicists do, as a matter of fact, hold that the change of velocity *is* continuous. The explanation of the apparent contradictions to the theory is to be found in the discontinuity of the physical processes correlated with the continuous series either of Time or of Space. For example, some of the surfaces in Space are boundaries separating contiguous bodies of quite different characters, such as solid and gaseous; here then, discontinuity holds of the dependent variable 'physical character' as determined by the independent variable 'spatial position.' Thus Time and Space are conceived as continuous in the first sense of the term, but the variations correlated with these independent variables are frequently discontinuous; and when this is the case discontinuity of the variations correlated with the variation of Time is explained by the discontinuity of the variations correlated with the variation of Space.

It should be observed that when speaking of change in the surface of a body, one part of which is solid and another part liquid, or one part rough and another part smooth, or one part red and another part green, the word change is applied to variations correlated with variation of Space instead of Time. We are apt to regard the words variation and change as synonymous, but it is very important to restrict the term variation to uncorrelated differences, and to apply the term change to differences correlated either with differences of Time or with differences of Space. This distinction affords some explanation of the rather vague statement of philosophers that causal process is continuous: a causal process is a process of change, and as such is correlated with a variation of Time or of Space, Time and Space being admittedly continuous; but the variations themselves in the changing variable correlated with Time or Space may be discontinuous. To elucidate this seemingly paradoxical notion of discontinuous variation, let us imagine a process of change the beginning and end of which are dated by the numbers 1 and 3. If the symbol *a* represents the determinate character dated at 1, and the symbol *f* the determinate character manifested at date 3—*a* and *f* denoting different determinates under the same determinable, whose determinates *a*, *b*, *c*, *d*, *e*, *f*, ... can be ranged in an order depending upon a comparison of degrees of difference—then the correlated discontinuity is illustrated by assuming that during the period from 1 to 3, the stretch from *c* to *e* (containing *d*) is *not* manifested. Through the period from date 1 to date 2, we will say, the manifestation changes continuously from the character *a* to the character *c*; and through the period from date 2 to date 3, it changes from *e* to *f* also continuously. At no instant within the period from 1 to 3 is the character *d* manifested. At the terminal phase belonging to the period 1 to 2, the manifestation must be characterised as *c*; but at the initial phase belonging to the period 2 to 3, it must be characterised as *e*. As so regarded the dates of these two phases cannot be identified; hence we cannot speak of *any* determinate character as being manifested *at* the date 2. On this ground therefore, at the instant in question, discontinuity of variation must be attributed to the process. A similar illustration, with Space instead of Time as the continuous variable, is afforded by supposing the surface of a table divided into two contiguous parts, one of which is entirely red, and the other entirely green, and considering the colour of the line which is the common boundary of these two parts. This question can perhaps hardly be said to involve a paradox, for colour characterises a *surface* and the surface-parts of a surface, it does not characterise any line which is contained in—but is not a part of—the surface. If this solution of the paradox be accepted, by the same method the paradox involved in correlation with Time may be removed; for, just as the parts of a surface which are two-dimensional as is the whole surface, must be distinguished from the boundary or separating lines which are contained in, but are not parts of, the surface; so the parts of Time which are of *one* dimension (as is the whole Time) must be distinguished from the instants, which are of *no* dimension, and which are contained in (but are not parts of) Time. And, just as colour was said to characterise the surface-parts of the table and not the lines contained in these surface-parts, so acceleration must be taken to characterise a movement occupying a certain part of Time, and not a *position* of the moving body which is correlated with an instant; for an instant is contained in, but is not part of, the period of Time to which the process of change is referred.

§ 4. Correlated continuity is probably always exhibited by immanent processes, i.e. these are non-discontinuous in their correlation with Time. The philosophical problem arises when transeunt causality is introduced. It may be stated thus: How is it that at a certain moment of time, two separate processes which have been immanently determined previously to this moment of time, cease to be for the subsequent time determined by merely immanent causality? This can only be explained by supposing some kind of connectional determination; i.e. if S and T represent the two immanent processes, in order to account for transeunt action taking place at one instant of time rather than at another, we must suppose some kind of connection between the manifestations of *S* and the manifestations of *T*; and the postulate is constructed that where there is transeunt action of *T* upon *S*, there is involved in the formula of determination a different mode of connection from that of Time. To this form of connection we give the name spatial, because in the case of physically determined phenomena transeunt causality always does operate in Space. Defined relations of spatial connection enter only in the formulae of interphysical causality, whereas every kind of causal formula involves defined relations of temporal connection. In the chapter on transeunt and immanent causality I have suggested that, for interpsychical causality, what takes the place of spatial connection is the attachment of both feeling and cognition to the same object; including, under the term feeling, conation in its two forms of attraction and repulsion felt with greater or less intensity towards experiences perceptually or imaginatively apprehended; and under the term cognition, including the variations in degrees of determinateness, as well as of *content*, according as the experience is thought of as having this or that character. Thus in my view the formula of interpsychical causality, introducing variable relations of feeling and cognition as causal determinants, takes the place of variable spatial relations as causal determinants in interphysical causality. For simplification of this exposition I have supposed the transeunt action to operate *ex abrupto* so that the instant of time at which it is dated can be determinately assigned. But our account of transeunt action must be extended to the cases where *some* spatial relation, which either remains unchanged or alters in the course of time, continually subsists between the manifestations of *S* and *T*, and where transeunt action is therefore temporally continuous instead of being instantaneous.

§ 5. We may now turn to the specific topic of this chapter, and consider the temporal relation of cause to effect which is commonly said to be that of before to after. In the first place it must be pointed out that manifestations cannot be related merely under the form of before to after, but must always, in addition, be regarded as manifestations of the same continuant-entity, whose nature is expressed in the formula according to which the preceding manifestations determine the succeeding. And, in the second place, parallel with the temporal order amongst the manifestations of a continuant, we have to consider the spatial order amongst the manifestations of an occupant. In somewhat figurative language we may conceive of an occupant as manifesting itself in a succession of regions which form rings from the more inner to the more outer, separated by concentric boundaries. The relations of the inner to the outer manifestations of an occupant are analogous to the relations of the preceding to the succeeding manifestations of a continuant. An adequate knowledge of the immanent nature of the entity would enable us to infer equally from the preceding to the succeeding, as from the succeeding to the preceding, in the case of a continuant; and from the inner to the outer, as from the outer to the inner in the case of an occupant. Thus reference to changing and spreading characters to the same continuant or occupant is the basal principle underlying causality. Now in spite of this possibility of reciprocal inference, we nevertheless regard the preceding as objectively determining the succeeding, as well as the inner as objectively determining the outer; and never reversely, the succeeding as objectively determining the preceding, and the outer as objectively determining the inner. The explanation of this refusal to reverse the order of objective determination in the temporal and spatial manifestations of continuants and occupants requires us, I think, to pass from immanent to transeunt causality. Thus, at a certain moment of time, an immanent process of causality may be broken in upon from without by an influence which modifies the succeeding manifestations, so that these are different from what they would have been under the uninterrupted course of immanent process. So while the manifestations which preceded the interruption can be determined from the mere knowledge of the formula of immanent causality, after the interruption the relation of the succeeding to the preceding is objectively differentiated from that of the preceding to the succeeding. It is to be noted that the temporal relation involved in transeunt causality itself, is neither that of succeeding nor of preceding, but of simultaneity. Analogously, in the case of the occupant, the reason why we regard the inner manifestations as objectively determining the outer, and not reversely, is explained by the introduction of transeunt causality. Just as, in time, we can take the boundary between the preceding and succeeding phases, and show that when a cause from without operates at this boundary, the succeeding phases are immediately modified; so we may take the surface of an occupant as the boundary at which a cause from without operates, and show that it immediately modifies the outer and not the inner state of the occupant. Simple illustrations of this principle are afforded by an extensible string, or a compressible fluid: when a string is subjected to an equal tension at both ends, the transeunt causality is from the extremes to the centre, while the immanent causality, which reacts, is from the centre to the extremes; or again, when a fluid is subjected to equal pressure throughout its surface, the transeunt causality is from the outer surface to the inner, while the immanent causality with which the fluid reacts is from the inner to the outer.

§ 6. The above account requires some explanatory modification; for in all such cases as those we are considering, manifestations of an occupant which are *actual* over certain regions of space, at any given time, are *potential* over other regions; and similarly, manifestations of a continuant which are actual throughout certain periods of time, are potential throughout the other periods. This point is illustrated with peculiar significance in psychology, where periods of apparent abeyance in consciousness of the familiar phenomena of association illustrate in the extremest sense the potentiality for manifestations. The occasions when this potentiality is converted into an actuality are when an experience breaks in upon the previous current of thought and operates transeuntly in modifying the subsequent processes. In a precisely similar way, the occasions when the potentiality of a body for exerting pressure or sustaining tension is converted into actuality, are those when it comes into transeunt contact with a foreign body which modifies its subsequent states. In any case of this kind we may distinguish those manifestations which are modified by the transeunt action from those which could have been determined without knowledge of such action. It will be found that the unmodified manifestations of the continuant are related to the modified as earlier to later, and of the occupant as inner to outer. Thus to take the occupant, for example, when a foreign body attracts a given body as a whole, it does not affect the internal motions of its parts, represented by temperature, chemical constitution, inner strains and stresses, etc., but only its situation relatively to other bodies, and these may be properly called outer manifestations relatively to the inner and immanent processes of the body. Contrasting an illustration of this kind with such transeunt action as the application of heat to a gas, the transeunt causality in the latter case appears to produce effects in the inner region as well as the outer occupied by the substance; but this is because the gas does not in truth constitute a unit-entity, and must be broken up into parts before we can apply with significance the distinction between the immanent and the transeunt. From the point of view of mechanical and thermal analysis the parts into which the gas must be broken up are molecules whose only inner and immanent manifestations are chemical. The application of heat affects the actions between the molecules themselves, as represented by their relative movements and mutual pressures, and these illustrate transeunt causality, the chemical or inner processes of the molecules being left unaffected to follow their own immanent course. The case of the gas, then, when properly analysed, is a further illustration of the principle that the transeunt processes modify the outer and the later manifestations without affecting the inner and the earlier; and that between the transeunt cause and the transeunt effect there is temporal simultaneity and spatial coincidence.

§ 7. The above illustration of transeunt and immanent action suggests a third kind of causality which requires separate consideration, viz. that involved in the compression of a compressible solid, as distinguished from the compression of a gas. Here the correlated conceptions of stress and strain are properly applied; a liquid or a solid when it is unnaturally compressed, exerts a force of expansion which decreases from a certain maximum to the minimum zero, as the compression is allowed to decrease. And a solid or extensible string, when it is unnaturally extended exerts a force of contraction which decreases from a certain maximum limit to the minimum zero as the extension is allowed to decrease. It will help us to understand the nature of the force of tension illustrated by the string if we contrast it with the force of attraction; for, while between two attracting bodies the force of approach is stronger the nearer they are to one another, between two parts, say, of a string the force of approach is stronger the further they have been pulled from one another. Now to understand the type of causality operating in the case of the compressed solid, we may mentally divide the volume which it occupies into an inner core and an outer ring. The effect of the pressure operating from the foreign force is to unnaturally contract the volume occupied by the inner core, causing an outward pressure upon the inner side of the outer ring. Apart from the outward pressure, the effect of the inward pressure would be manifested in the restoration of the outer ring to its natural size; we may therefore properly take the contracted condition of the inner core as the immanent cause of the subsequent expansion of the outer ring. This form of causality illustrates the same type of analogy between temporal and spatial factors as we have already noted, the inner in space corresponding to the earlier in time, and the outer in space corresponding to the later in time, while the causality operating at the common boundary between the inner core and the outer ring corresponds to the moment of time at which the condition of the inner core influences the condition of the outer ring. If then immanent causality alone were involved, our knowledge of the shape and size of the inner core would determine for us a knowledge of the subsequent and contiguous shape and size assumed by the outer ring. But when the transeunt causality from the foreign force is brought into consideration, the subsequent and contiguous shape and size assumed by the outer ring is modified. Having divided at an arbitrary surface the inner core from the outer ring, we must make a correspondingly arbitrary division in time between the earlier and the later states of the body. Considering the solid body alone, the inner core is first under a pressure dependent upon its unnatural shape and size, and the subjection of the outer ring to the foreign compressing force of pressure is later in time. So the inner surface of the outer ring at the earlier stage is pressing outwards, and the outer surface of this outer ring at the later stage is pressing inwards. Hence the pressure at the inner surface at the earlier stage represents that part of the process (determined solely by immanent conditions) which is unmodified by transeunt action; while the pressure at the outer surface at the later stage represents that part of the process which is modified by transeunt action. The case of the extensible string is capable of precisely similar analysis; so also is that of the varying temperature of gas enclosed in an envelope. In all these cases, the immanent tendencies operate in the direction of an assignable form of equilibrium, and by dividing the whole process into temporal and spatial parts corresponding to one another we shall always find, by taking the earlier stage to correspond with the inner region, and the later stage with the outer region, that the former represents the part of the process unmodified, and the latter the part of the process modified by transeunt action.

A failure in the analogy between Space and Time hitherto unnoted, may here be pointed out. Whereas the dating of a process is absolute, in the sense that it is independent of the continuant to which the process refers, the locating of a process as being relatively inner or outer is not absolute, for what is relatively inner to one occupant is relatively outer to another. To establish the required analogy, it would be necessary to conceive that, of two temporally distinguished parts of a process extending through Time, that which is earlier when referred to one continuant is later when referred to another; just as, of two spatially distinguished parts of a process extending through Space, that which is inner when referred to one occupant is outer when referred to another.

APPENDIX ON EDUCTION

§ 1. In the problem before us we shall be concerned with a certain adjectival determinable P which has α determinate values—p_1, p_2, … p_α—and shall proceed to consider M instances, each of which is characterised by one or other of these α determinate characters.

Any actual set of occurrences of length M will exhibit a certain proportion among the α determinate characters;—m_1 occurrences of p_1, m_2 of p_2 … m_α of p_α (say), where

$$m_1 + m_2 + \ldots + m_\alpha = M.$$

The proportion $m_1 : m_2 : \ldots : m_\alpha$ exhibited in M occurrences will be denoted by μ.

These occurrences also will be presented in a definite order.

The order in which the occurrences exhibiting the proportion μ are presented will be denoted by $\overrightarrow{\mu}$.

The following two elementary arithmetical formulae will be required:

(1) *Combination-formula.*

The number of integral solutions of the equation

$$m_1 + m_2 + \ldots + m_\alpha = M,$$

i.e. the number of values that μ may assume in M occurrences, is

$$\frac{\alpha(\alpha+1)\ldots(\alpha+M-1)}{M!}.$$

(2) *Permutation-formula.*

The number of different orders $\overrightarrow{\mu}$, in which a given proportion $m_1 : m_2 : \ldots : m_\alpha$ may be presented in M occurrences, is

$$\frac{M!}{m_1!\,m_2!\ldots m_\alpha!}.$$

§ 2. Probability is a magnitude to be attached to any possibly true or possibly false proposition; not, however, to the proposition in and for itself, but in reference to another proposition the truth of which is supposed to be known. For example, the probability of the proposition that 'The next throw of a certain coin will yield head' may have its value assigned by the knowledge that 'It will yield either head or tail.' The value of the probability as so determined is not necessarily the same as that determined by the knowledge that 'The previous throws of the coin have presented heads and tails with a certain frequency.' The proposition to which the probability attaches will be conveniently termed the *proposal*; and the proposition to which the probability refers as that whose truth is supposed to be known will be conveniently termed the *supposal*.

Furthermore, in order that the notion of probability shall have significance, it is requisite that the proposition standing as supposal *shall not be known to be false*. Using the notation adopted by Mr J. M. Keynes, which introduces the solidus:

p/s symbolises the probability of the proposal p as depending upon or referred to the supposal s.

The notation p/s may be read 'p upon s' or 'p given s.'

The maximum limiting magnitude of p/s is certitude: viz. when the truth of p is implied by s. Its minimum limiting magnitude is contra-certitude: viz. when the falsity of p is implied by s. Since probability-values are signless, the minimum value (contra-certitude) must always be represented as *zero*; and, since certitude is the maximum probability-value, all other probability-values are (proper) fractions of certitude. It is, in fact, usual to express probability-values as pure fractions, such as $\frac{1}{2}$ or $\frac{2}{3}$; and to express certitude by unity. But this representation is logically false, and should only be permitted as a convenient abbreviation.

In estimating the probability of p as depending on the specific knowledge s it is essential that s should represent the *whole* of the supposed knowledge, relevant to the case. Briefly, the dependence indicated by the equation $p/s = \frac{2}{3}$ of certitude (say), when expressed as an implication, means:

If s *alone* were known, then the probability of p would be $\frac{2}{3}$ of certitude.

If t also were known, p/st would not necessarily be the same.

In this respect, the relation of dependence for probability is to be contrasted with the relation of implication. Thus

'p is implied by s' corresponds to $p/s =$ certitude.

Now, if 'p is implied by s,' then also 'p is implied by st' and hence p/st also = certitude.

In other words, additional supposed knowledge cannot alter the degree of probability of any proposition known to be true or to be false, but it may always alter the degree of probability of a proposition not known to be true or to be false.

§ 3. Two axioms are required for the working of the probability-calculus: viz. the additive and the multiplicative. With the notation above explained, these axioms may be formulated as follows:

Additive axiom:

If p and q are known to be not both true, then

$$(p \text{ or } q)/h = p/h + q/h.$$

Multiplicative axiom:

If p is not known to be false, then

$$(p \text{ and } q)/h = p/h \times q/(p \text{ and } h).$$

When such symbols as p, q stand for propositions, the conjunctive 'p and q' will be abbreviated into pq. But, when x, y (say) stand for quantities, then $x \,.\, y$ or xy will mean '$x \times y$.' On the other hand, 'p or q' should never be written '$p+q$'; nor should p/q be written $p \div q$ or $\frac{p}{q}$ (in spite of certain analogies).

Thus the formula for multiplication may be written

$$pq/h = p/h \times q/ph.$$

§ 4. The following corollaries will be required in the sequel:

Cor. 1. If p_1 or p_2 or $\ldots p_r \equiv p$, where $p_1, p_2, \ldots p_r$ are co-disjunct, then, by additive axiom,

$$p/h = p_1/h + p_2/h + \ldots + p_r/h.$$

Cor. 2. If, further,

$$p_1/h = p_2/h = \ldots = p_r/h,$$

then each of these $= \frac{p/h}{r}$.

Cor. 3. If 'q implies p,' i.e. $q \equiv p$ and q, then

$$q/h = (p \text{ and } q)/h = p/h \times q/ph,$$

by multiplicative axiom.

COR. 4. If s_1 or s_2 or $\dots s_r \equiv s$, where $s_1, s_2, \dots s_r$ are co-disjunct, and if, further,

$$p/s_1h = p/s_2h = \dots = p/s_rh,$$

then each of these $= p/sh$.

For, let each of the above $= x$. Then

$$ps/h = ps_1/h + ps_2/h + \dots + ps_r/h,$$

i.e. $$s/h \,.\, p/sh = s_1/h \,.\, p/s_1h + s_2/h \,.\, p/s_2h + \dots + s_r/h \,.\, p/s_rh$$
$$= (s_1/h + s_2/h + \dots + s_r/h)\, x$$
$$= s/h \,.\, x.$$
$$\therefore\ x = p/sh. \qquad \text{Q.E.D.}$$

§ 5. Now the axioms of probability enable us to infer any probability-conclusion *only* from probability-premisses. In other words, the calculus of probability does not enable us to infer any probability-value unless we have some probabilities or probability relations *given*. Such data cannot be supplied by the mathematician. E.g. the rules of arithmetic and the axioms of the probability-calculus are utterly impotent to determine, on the supposed knowledge that the throw of a coin must yield either head or tail and cannot yield both, the probability that it will yield head or that it will yield tail. We must assume that the two co-exclusive and co-exhaustive possibilities are *equally probable*, before we can estimate the probability of either as being a half of certitude. The assumptions ultimately required must be regarded as *Postulates*, and their critical examination will not here be entered upon. The working postulates are such as assert equiprobability amongst alternative possibilities; and constitute what may be called, in Mr Keynes's terminology, *postulates of indifference*.

The precise form of the postulate required in each particular application must be justified by the special nature of the case. We shall immediately lay down the two postulates employed in the theory of eduction, postponing for the present any further philosophical discussion.

§ 6. The two following postulates in the Theory of Eduction are concerned with the possible occurrences of the determinates $p_1, p_2, \ldots p_a$ under the determinable P. The symbols of § 1 are employed.

(1) *Combination-Postulate.*

In a total of M instances, any proportion, say $m_1 : m_2 : \ldots : m_a$, where $m_1 + m_2 + \ldots + m_a = M$, is as likely as any other, prior to any knowledge of the occurrences in question.

(2) *Permutation-Postulate.*

Each of the different orders in which a given proportion $m_1 : m_2 : \ldots : m_a$ for M instances may be presented is as likely as any other, whatever may have been the previously known orders.

In what follows certitude will be represented by unity.

By (1), The probability of any one proportion in M instances

$$= \frac{M!}{a\,(a+1)\,(a+2)\ldots(a+M-1)}.$$

By (2), The probability of any one permutation in which the proportion $m_1 : m_2 : \ldots : m_a$ in M instances may be presented

$$= \frac{m_1!\,m_2!\,m_3!\ldots m_a!}{M!}.$$

§ 7. Adopting the notation explained above, these postulates may be symbolised:

$$(1) \qquad \mu/h = \frac{M!}{\alpha(\alpha+1)\ldots(\alpha+M-1)}.$$

$$(2) \qquad \vec{\mu}/\mu h = \frac{m_1!\,m_2!\ldots m_\alpha!}{M!}.$$

Now (see Cor. 3)

$$\vec{\mu}/h = \mu\vec{\mu}/h = \mu/h \times \vec{\mu}/\mu h,$$

$$\therefore \ (3) \qquad \vec{\mu}/h = \frac{m_1!\,m_2!\ldots m_\alpha!}{\alpha(\alpha+1)\ldots(\alpha+M-1)}.$$

Formula (3) gives the prior probability that, in a set of M instances, the characters $p_1, p_2, \ldots p_\alpha$ under P shall occur in a determinately assigned sequence in which the proportion and the permutation of these character-manifestations are both fixed.

Taking N instances (next following the M instances) presenting the proportion $n_1 : n_2 : \ldots : n_\alpha$, where $n_1 + n_2 + \ldots + n_\alpha = N$, the principle of formula (3) may be extended to $M + N$ instances.

Thus

$$(4) \quad \overrightarrow{\mu+\nu}/h = \frac{(m_1+n_1)!\,(m_2+n_2)!\ldots(m_\alpha+n_\alpha)!}{\alpha(\alpha+1)\ldots(\alpha+M+N-1)}.$$

Now

$$\overrightarrow{\mu+\nu}/h = (\vec{\mu} \text{ and } \vec{\nu})/h = \vec{\mu}/h \times \vec{\nu}/\vec{\mu}h, \quad \text{by Mult. axiom.}$$

$$\text{Hence (5) } \vec{\nu}/\vec{\mu}h = \frac{\overrightarrow{\mu+\nu}/h}{\vec{\mu}/h} = \frac{(m_1+n_1)!}{m_1!} \ldots \frac{(m_\alpha+n_\alpha)!}{m_\alpha!}$$

$$\times \frac{1}{(M+\alpha)(M+\alpha+1)\ldots(M+N+\alpha-1)}$$

by (3) and (4).

From formula (5) which gives $\vec{\nu}/\vec{\mu}h$ we proceed to find the value of $\nu/\mu h$ which will complete our solution.

Let $\vec{\nu}_a$, $\vec{\nu}_b$, $\vec{\nu}_c$, etc. represent the different possible permutations of the same proportion $\nu = n_1 : n_2 : \ldots : n_\alpha$. The number of these is $\dfrac{N!}{n_1!\, n_2! \ldots n_\alpha!}$.

Now expression (5) is independent of the *orders* $\vec{\nu}_a$, $\vec{\nu}_b$, etc. Hence

$$\vec{\nu}_a / \vec{\mu} h = \vec{\nu}_b / \vec{\mu} h = \vec{\nu}_c / \vec{\mu} h = \text{etc.}$$

Hence, by Cor. 2,

$$(6) \quad \nu / \vec{\mu} h = (\vec{\nu}_a \text{ or } \vec{\nu}_b \text{ or } \vec{\nu}_c \text{ or etc.}) / \vec{\mu} h$$

$$= \frac{N!}{n_1!\, n_2! \ldots n_\alpha!} \cdot \vec{\nu}_a / \vec{\mu} h$$

$$= \frac{N!}{(M+\alpha)(M+\alpha+1)\ldots(M+N+\alpha-1)} \times \frac{(m_1+n_1)!}{m_1!\, n_1!} \cdots \frac{(m_\alpha+n_\alpha)!}{m_\alpha!\, n_\alpha!}.$$

Again, let $\vec{\mu}_a$, $\vec{\mu}_b$, $\vec{\mu}_c$, etc. represent the different possible permutations of the proportion

$$\mu = m_1 : m_2 : \ldots : m_\alpha.$$

Now expression (6) is independent of the *orders* $\vec{\mu}_a$, $\vec{\mu}_b$, etc. Hence

$$\nu / \vec{\mu}_a h = \nu / \vec{\mu}_b h = \nu / \vec{\mu}_c h = \text{etc.}$$

Hence, by Cor. 4,

$$(7) \quad \nu / \mu h = \nu / (\vec{\mu}_a \text{ or } \vec{\mu}_b \text{ or } \vec{\mu}_c \text{ or etc.})\, h$$

$$= \nu / \vec{\mu}_a h$$

$$= \frac{N!}{(M+\alpha)(M+\alpha+1)\ldots(M+N+\alpha-1)} \times \frac{(m_1+n_1)!}{m_1!\, n_1!} \cdots \frac{(m_\alpha+n_\alpha)!}{m_\alpha!\, n_\alpha!}.$$

This provides the required formula, viz.:—

The probability of any *proposed* proportion in N unexamined cases as depending upon any *supposed* proportion in M examined cases.

It will be observed that the highest value of this probability, if $N = M$, is given by

$$n_1 = m_1;\ n_2 = m_2;\ \ldots\ n_a = m_a,$$

i.e. the most likely proportion for the new cases is the proportion holding of the known cases.

And, generally, the more closely ν agrees with μ, the greater is the probability that ν will be true when μ is known to be true.

§ 8. Elucidation of the formula for $\overrightarrow{\nu}/\mu h$.

As above, we see that $\overrightarrow{\nu}/\mu h = \overrightarrow{\nu}/\overrightarrow{\mu} h$.

Taking N to be successively 1, 2, 3, etc. the simplicity of the above results will be readily seen.

Thus, for $N = 1$, the different values of the proposal $\overrightarrow{\nu}$ are $p_1, p_2, \ldots p_a$. Thus

$$p_1/\mu h = \frac{m_1 + 1}{M + a};\ p_2/\mu h = \frac{m_2 + 1}{M + a};\ \ldots p_a/\mu h = \frac{m_a + 1}{M + a}.$$

For $N = 2$, the different values are the dual permutations $p_1 p_1;\ p_1 p_2;\ \ldots p_1 p_a;\ p_2 p_1;\ p_2 p_2;\ \ldots p_2 p_a$, etc., etc. Thus

$$p_1 p_1/\mu h = \frac{(m_1 + 1)(m_1 + 2)}{(M + a)(M + a + 1)};$$

$$p_1 p_2/\mu h = \frac{(m_1 + 1)(m_2 + 1)}{(M + a)(M + a + 1)}, \text{ etc.,}$$

$$p_2 p_1/\mu h = \frac{(m_2 + 1)(m_1 + 1)}{(M + a)(M + a + 1)};$$

$$p_2 p_2/\mu h = \frac{(m_2 + 1)(m_2 + 2)}{(M + a)(M + a + 1)}, \text{ etc.,}$$

etc., etc., etc.

For $N=3$, the probabilities of the triple permutations are:

$$p_1 p_1 p_1/\mu h = \frac{(m_1+1)(m_1+2)(m_1+3)}{(M+\alpha)(M+\alpha+1)(M+\alpha+2)};$$

$$p_1 p_1 p_2/\mu h = \frac{(m_1+1)(m_1+2)(m_2+1)}{(M+\alpha)(M+\alpha+1)(M+\alpha+2)}, \text{ etc.,}$$

etc.

By addition of the values for $N=2$, we obtain those for $N=1$. And, by addition of the values for $N=3$, we obtain those for $N=2$. And so on. In this way the correctness of each formula is verified.

Moreover, all specific results of the formula giving $\vec{\nu}/\mu h$ may be schematised—if we typify occurrences as draws from a bag containing an indefinite number of balls of the different colours $p_1, p_2, \ldots p_\alpha$—by supposing a model bag containing at first α balls of different colours. As each new ball is drawn from the *real* bag, its colour is observed and it is transferred to the *model* bag. Then, *the probability of any proposed colour being drawn from the* real *bag is the same as that of its being drawn from the* model *bag.*

§ 9. The type of case for which the two Postulates are permissible may be thus described.

It is known that there are certain conditions which are *constant* in all the occurrences that may take place and to which our observations and inferences refer. It is also presumed that these permanent or constant conditions are such as *tend* to produce a certain (but unknown) proportion amongst the manifested characters within any given set of M occurrences. On the other hand, each individual occurrence is actually occasioned by *variable* conditions, which are causally independent of one another, and are such that no prediction as to their result in any one case can be made.

In such a typical state of affairs, what is unknown is the proportion which tends to be exhibited owing to the unchanged or permanent set of causal conditions. While, therefore, the determinate issue in any set of instances is *causally* independent of what has previously occurred, yet it is *epistemically* dependent; i.e. from the point of view of knowledge, the observation of previously examined instances rationally influences our estimate of probability in regard to what will subsequently occur.

The first postulate, that (in M cases) one *proportion* is as likely as another, is negatively justified by our ignorance of the proportion which the permanent conditions tend to produce. And our second postulate, that one *order* in which any proportion may be manifested is as likely as any other, is positively justified by our knowledge that the variable conditions which occasion each individual occurrence are ontologically independent of those which occasion any other individual occurrence.

When it is said that the permanent conditions *tend* to produce a certain proportion $p_1 : p_2 : \ldots : p_\alpha$, by this it is not meant that such a proportion will be more nearly approached as the series is indefinitely prolonged. For, on the contrary, in $2M$ cases the proportion $p_1 : p_2 : \ldots : p_\alpha$ is very much less likely to be exhibited than in M cases; since the number of arithmetically possible proportions is much greater in a total of $2M$ than in a total of M.

We may finally point out that the type of case for which our theory of eduction holds may be figuratively represented by imagining a die, in the form of a solid polyhedron, whose plane faces are not more than α in number. Moreover, the die is not known to be either physically or geometrically regular. Each throw of the die represents an occurrence; and, according as the die falls *upon* one or another plane face, we represent the occurrence as being characterised by one or another of the a possible determinate adjectives—$p_1, p_2, \ldots p_a$.

The constancy of the physical and geometrical properties of the die corresponds to the *constancy* of those unchanged causal conditions upon which the occurrences depend; while the variable and unassignable impetus of each toss corresponds to the *variable* condition which occasions the actual issue in each individual occurrence:—the varying condition which determines the issue in any one case being causally independent of that which determines the issue in any other case.

INDEX

Actual and potential manifestations 87
Adjective, discrimination of 53
Agent and patient xxiii, 69, 86
Agreement, use in induction 24, 28; modification of figure 61; and variancy 37, 48, 51
Analogy, relation to induction 46
Analysis of instances 24
Arithmetic and logic xiv
Article, *see* Predesignations
Attention, its effect and cause 111, 158; and freewill 123

Bacon's 'Table of Degrees' 21
Body and mind xxvi, 151, 159

Cause, agents 69, 86; Aristotelean classification 73; and conation 120, 150; convergence of factors 143; efficient 139; and events 69; and foreknowledge 109, 118; and implication 4, 144; parallel processes 144; and properties 71; temporal paradox of 70
Causes and effects, complete assignment of 56, 63; 'different' defined 59; distinction between 79, 132; homogeneity 69; plurality of 54; reciprocity of 56, 62; spatio-temporal relations between 79, 132
Change, and cause 66, 76, 85; a complex notion 156; continuity of 162; and the determinable 84; and continued existence 68, 80, 100; discontinuity in 166; modern views on 100
Characterisation and inherence 66
Cognition, and conative deliberation 124; and final cause 139, 159; in interpsychical causality 170; and neural process 105, 113
Complementary universals 62
Conation, a form of causality 118; and judgments of value 126
Concrete and abstract 1
Contingency, nomic 10
Continuant, and causal agency 86; and change 81; concept criticised 101; defined 98; and the determinable 85; identity of 79, 128; and immanent causality 97, 128; and law 82; relation to occurrent xviii, 66, 95; physical illustration of 78; psychical illustration of 82; psychical and physical contrasted xxii, 137; and substance 80; relation to substantive 67; and systems of sub-continuants 92
Continuity and discreteness 162; and discontinuity 165, 167
Convergent cause process 143; illustrated 147
Conversion, fallacy of 54

Determinable, and change 84; and the continuant xix, 67, 85; and the given 3
Determinism and freewill xxxiii, 123
Difference, and independence 38; use in induction 25, 29
Discrimination 53
Divergent effect process 143; psycho-physical illustration 149

Dualism xix; objections to xxxi

Eduction, defined 43; and induction 47
Effects, divergence of 143; parallel processes 144; plurality of 54; and properties 72
Effort, function of 107; types of 112
Ego, as agent 103; as continuant 136
Emotion, causal relation of factors 135, 152
Events and occurrents xxi; psychical and physical xxii; relations between 69, 143
Evidence, complementary 52
Evolution, and absolute time 22
Existents and co-existence 71
Experiment and natural law 18; and transeunt causality 129
Extensional aspects of eduction 46; wholes 101

Facts, the test of formulae 33; relations between 161
Factual determination 161; propositions 1, 3; universals 6, 11
Force, actuality of 87; stress and strain 175
Freewill xxxiii, 123
Functions and properties 97

'Given' and 'real' 3

Hume, on causality 4
Hypothesis, meanings of 30

Identity and otherness xv
Immanent causality, and the continuant 94, 97, 128; in physics 129; in psychology 102, 136; function in science 140; and systems of sub-continuants 92, 141; and time 177; and transeunt xxiv
Implication and cause 4
Independency and difference 38, 48, 51, 65; orders of 41
Induction, and analogy 46; criteria of 19; and eduction 47; use of experiment in 17, 39; hypothetical 30
Inference and divergent causality 151
Instantial premiss 44
Intensional aspects of eduction 46
Intermediary premiss 46, 50
Invariability of sequence xxxii

James, W., on conative conflict 122; on emotion 134
Jevons on induction 31

Kant on causality 4; on substance 98

Language, relation to thought 116
Law, and experiment 17; and factual universals 11; logical form of 4, 8; and uniformities of co-existence 70
Logic as philosophy xvi
Logical determination 161

Mathematics and logic xv
Matter, and mind 151, 159; primary qualities of 88; spatial forms of 91
Mill on causality 5, 70; on freewill xxxiv; on syllogism 43
Movement, and convergent causality 146, 153; logical account of 78, 127

Nomic necessity and contingency 9
Number xv; and order 163

Objective determination 4, 69, 172
Occupant 95, 171

Occurrent, and continuant 66, 78, 95; and event xxi; and transience 68
'Of,' meaning of xx
Order, serial 162

Parallelism, psycho-physical xxviii, 105, 112; diagram for 143, 156; illustrated 118, 134; and inferential determination 146, 148
Particulars, inference from 44
Parts and boundaries 163, 169
Permanence and change 68, 100; of substance 99
Plurality of Causes, defined 54; illustrated 60
Possible, meanings of 6; and potential 14; range of 13
Postulate defined xviii
Potentiality defined 14; and secondary qualities 88; and transeunt causality 174
Predesignations and reference 3
Probability and eduction 48; and evidential data 49
Problematic induction, principle of 34
Properties and cause 71, 86; relation to continuant 68, 96; formula for 86, 97; denoting potentialities 88, 138
Psychology, immanent process in 102, 139; transeunt process in 103, 173
Purpose xxviii

Separation 53; in fact and thought 164
Series, discrete and continuous 162
Space, order in 163
Specification, principle of 21
Spinoza xix
Subjective activity, and cause 103, 109; and effort 107; types of 111
Substance, Kant's views on 98; metaphysical notion of 80
Substantival identity 79; separation 53

'Thing' or continuant 98
Thought, *see* Cognition
Time and causality xxv, 70, 74, 171; and the continuant 67; order in 164; an independent variable 166
Time and Space, conditions of otherness 79; forms of nexus 90; not analogous 177; and transeunt causality 172; and correlated variables 167
Transeunt causality xxiii; relation to immanent xxiv; illustrated from physics 130, 174; in psychology 102, 136, 173; and psycho-physical process 104; function in science 141; and systems of sub-continuants 92, 141; and temporal relations 172

Uniformities, causal 94; of co-existence 70, 74, 94

Value, judgments of 125
Variables, discontinuous 165
Variancy and agreement 37, 48, 51
Verification 31

Wholes, extensive and extensional 164

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